

# Introduction to Trigonometry Key Concepts

## Right Triangle Trigonometry Lesson

### Trigonometric Ratios

$$\text{sine } \theta = \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine } \theta = \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent } \theta = \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{cosecant } \theta = \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{secant } \theta = \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{cotangent } \theta = \cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

### Cofunction Identities

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\cot \theta = \tan(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta)$$

$$\csc \theta = \sec(90^\circ - \theta)$$

## Solving Right Triangles Lesson

### Inverse Trigonometric Functions

Inverse trigonometric functions are used to find the missing angle. The inverse sine function, denoted  $\sin^{-1} x$  or  $\arcsin x$ , is defined as the following:

- $\sin^{-1} x = \theta$ , where  $x = \sin \theta$

The definition of each inverse trigonometric function is similar to the inverse sine function:

- $\cos^{-1} x = \theta$ , where  $x = \cos \theta$

- $\tan^{-1}x = \theta$ , where  $x = \tan \theta$

## Angle Measurements Lesson

### Angle Measurements

The relationship between radians and degrees is  $\theta = \frac{s}{r}$ , where  $\theta$  is the measure in radians of the central angle which intercepts an arc of length  $s$  for a circle of radius  $r$ .

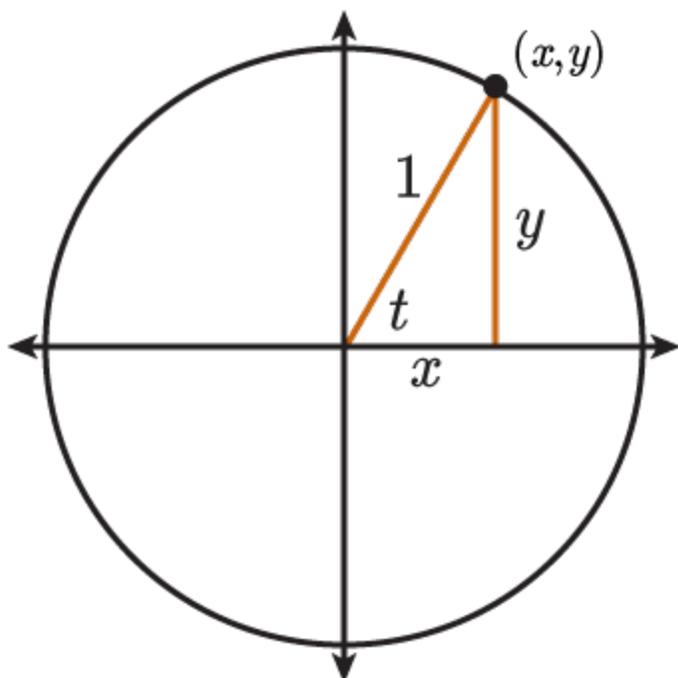
### Conversion Factors

- 1 radian =  $\left(\frac{180}{\pi}\right)^\circ$
- $1^\circ = \frac{\pi}{180}$  radians

## The Unit Circle Lesson

### Trigonometric Functions of Real Numbers

Let  $t$  be a real number radian measure of an angle in standard position on the unit circle, and  $(x, y)$  be the point at which the terminal side intersects the unit circle.

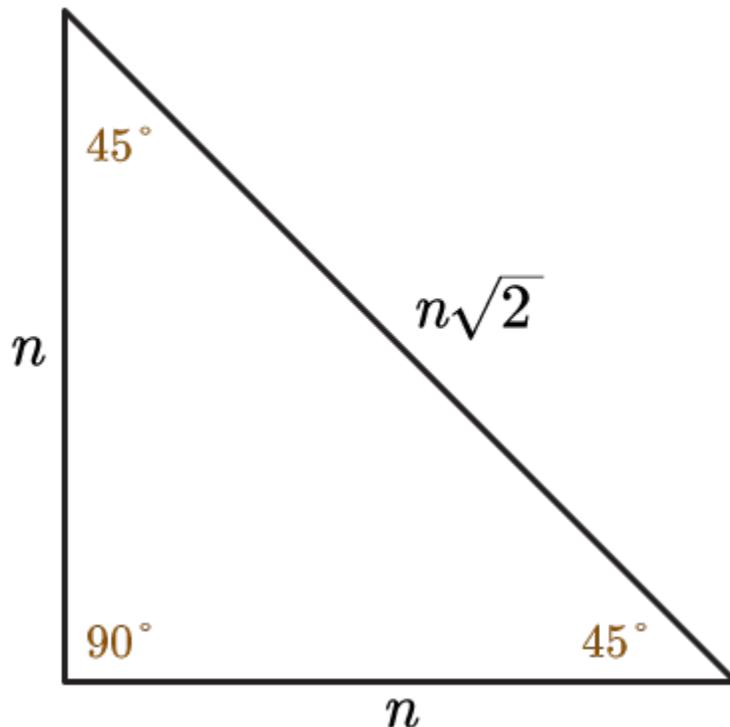


$\sin t = y$	$\cos t = x$	$\tan t = \frac{y}{x}, x \neq 0$
$\csc t = \frac{1}{y}, y \neq 0$	$\sec t = \frac{1}{x}, x \neq 0$	$\cot t = \frac{x}{y}, y \neq 0$

## Special Right Triangles Lesson

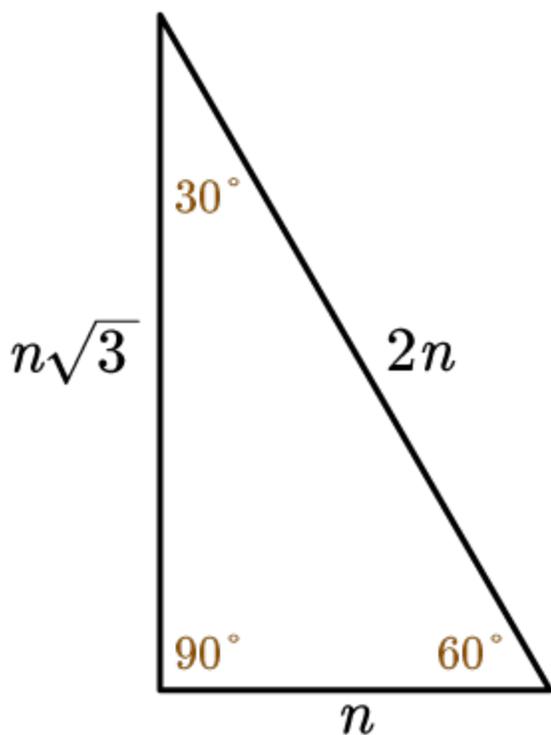
### 45-45-90 Triangle

A 45-45-90 triangle will always have the relationship between the side lengths as shown in the diagram. The legs are equal lengths while the hypotenuse is equal to the length of the leg times  $\sqrt{2}$ .



### 30-60-90 Triangle

A 30-60-90 triangle will always have the relationship between the side lengths as shown in the diagram. The hypotenuse is twice as long as the shortest leg. The long leg is equal to the length of the short leg times  $\sqrt{3}$ .



## Special Angles

$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ$	0	1	0
$30^\circ$ or $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45^\circ$ or $\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^\circ$ or $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ$ or $\frac{\pi}{2}$	1	0	undefined

# Trigonometric Functions in Quadrants I–IV

## Lesson

### Signs of Trigonometric Functions in Each Quadrant

Quadrant II		Quadrant I	
$\sin \theta +$	$\csc \theta +$	$\sin \theta +$	$\csc \theta +$
$\cos \theta -$	$\sec \theta -$	$\cos \theta +$	$\sec \theta +$
$\tan \theta -$	$\cot \theta -$	$\tan \theta +$	$\cot \theta +$
Quadrant III		Quadrant IV	
$\sin \theta -$	$\csc \theta -$	$\sin \theta -$	$\csc \theta -$
$\cos \theta -$	$\sec \theta -$	$\cos \theta +$	$\sec \theta +$
$\tan \theta +$	$\cot \theta +$	$\tan \theta -$	$\cot \theta -$

### Trigonometric Functions of Quadrantal Angles

Function	$0$ or $2\pi$ $0^\circ$ or $360^\circ$	$\frac{\pi}{2}$ or $90^\circ$	$\pi$ or $180^\circ$	$\frac{3\pi}{2}$ or $270^\circ$
$\sin \theta$	0	1	0	-1
$\cos \theta$	1	0	-1	0
$\tan \theta$	0	undefined	0	undefined
$\csc \theta$	undefined	1	undefined	-1
$\sec \theta$	1	undefined	-1	undefined
$\cot \theta$	undefined	0	undefined	0