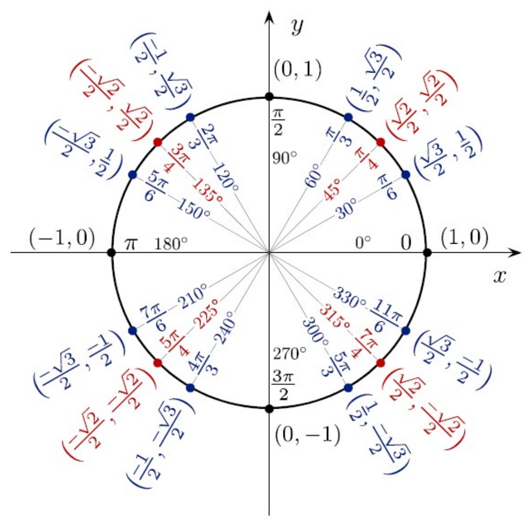
UNIT 3 LESSON 2

PRECALCULUS B

LESSON 2:

- Sum & Difference Formulas

We will need the Unit Circle:



SUM & DIFFERENCE FORMULAS

For when the angle you are given isn't one of the special angles on the Unit Circle.

. . . But you can make it by adding or subtracting with the angles on the Unit Circle!!

For example:
75° = 30° + 45°
15° = 45° - 30°
105° = 60° + 45°

Can you think of others?

Note: Yes, you could just use a calculator, but that is usually a rounded approximation. Using the Unit Circle gives you the exact answer!

SUM & DIFFERENCE FORMULAS

NOTE: sin(75) DOES NOT EQUAL sin(30) + sin (45)!!

sin(75) ≈ 0.965

sin(30) = 0.5

sin(45) ≈ 0.707

But 0.5 + 0.707 = 1.207 . . . NOT 0.965!!

sin(75) equals sin(30+45) & there is a rule for that

Because adding the angles is not the same as adding the trig ratios.

SUM & DIFFERENCE FORMULAS

Here are the Formulas:

Sum and Difference Formulas

1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

SUM & DIFFERENCE FORMULAS

Sum and Difference Formulas

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
5. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
6. $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Translation:
a is the Greek letter alpha
ß is the Greek letter beta
Both represent angles.

SUM & DIFFERENCE FORMULAS

What similarities or differences do you see in these formulas??

Sum and Difference Formulas

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
5. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
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SUM & DIFFERENCE FORMULAS

Sum and Difference Formulas

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Note . . . remember that tangent has asymptotes, so if the given angle is at an asymptote, then tangent is undefined and you can't use the tangent formulas!

SUM & DIFFERENCE FORMULAS

Back to using these formulas . . .

cos(75) = ??

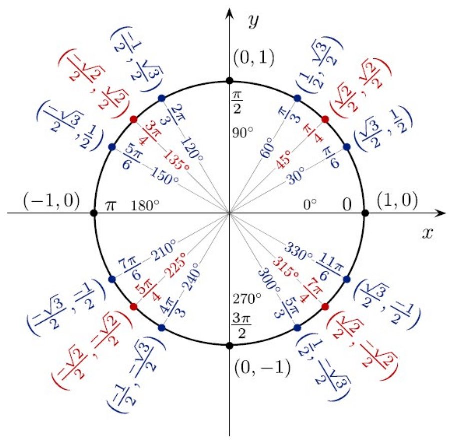
= cos(30 + 45)

= cos(30) · cos(45) – sin(30) · sin(45)

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

SUM & DIFFERENCE FORMULAS

cos(75) = cos(30) · cos(45) – sin(30) · sin(45)

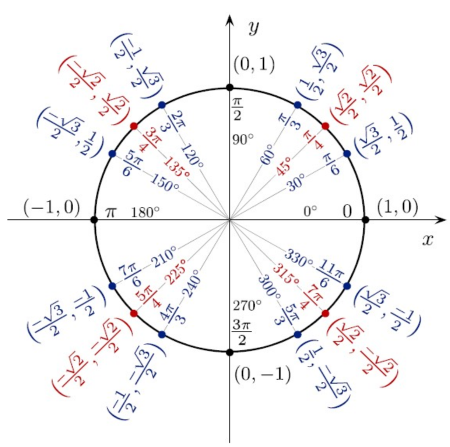


Remember,
(x, y) = (cos, sin)

cos 30 =
cos 45 =
sin 30 =
sin 45 =

SUM & DIFFERENCE FORMULAS

cos(75) = cos(30) · cos(45) – sin(30) · sin(45)



Remember,
(x, y) = (cos, sin)

cos 30 = √3/2
cos 45 = √2/2
sin 30 = 1/2
sin 45 = √2/2

SUM & DIFFERENCE FORMULAS

cos(75) = cos(30) · cos(45) – sin(30) · sin(45)

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

≈ 0.2588

Remember,
(x, y) = (cos, sin)

cos 30 = √3/2
cos 45 = √2/2
sin 30 = 1/2
sin 45 = √2/2

No, you can't subtract √6 - √2.
They are not like terms.
So this fraction answer is simplified.

SUM & DIFFERENCE FORMULAS

These also work with radians, of course, . . .

sin(π/12)

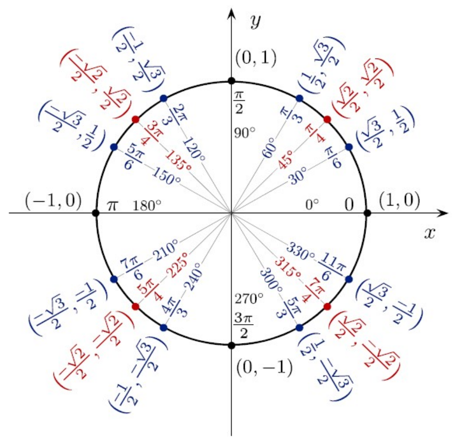
= sin(π/4 – π/6)

= sin(π/4)cos(π/6) – cos(π/4)sin(π/6)

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

SUM & DIFFERENCE FORMULAS

sin(π/12) = sin(π/4)cos(π/6) – cos(π/4)sin(π/6)



Remember,
(x, y) = (cos, sin)

sin(π/4) =
cos(π/6) =
cos(π/4) =
sin(π/6) =

SUM & DIFFERENCE FORMULAS

sin(π/12) = sin(π/4)cos(π/6) – cos(π/4)sin(π/6)

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Remember,
(x, y) = (cos, sin)

sin(π/4) = √2/2
cos(π/6) = √3/2
cos(π/4) = √2/2
sin(π/6) = 1/2

SUM & DIFFERENCE FORMULAS

sin(π/12) = sin(π/4)cos(π/6) – cos(π/4)sin(π/6)

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

cos(75) = cos(30) · cos(45) – sin(30) · sin(45)

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Huh . . .

Why are these the same?

Cofunctions of complementary angles are equal!

sin(π/12) = sin(π/4 - π/6) = sin(45° – 30°) = sin(15°)

cos(75°) = cos(30° + 45°)

15° + 75° = 90° … so they are complementary angles!

Which means sin(15°) = cos(75°)

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

COFUNCTION IDENTITIES

Cofunction Identities, radians		Cofunction Identities, degrees	
$\sin\left(\frac{\pi}{2}-x\right)=\cos x$	$\cos\left(\frac{\pi}{2}-x\right)=\sin x$	$\sin(90^\circ - x)=\cos x$	$\cos(90^\circ - x)=\sin x$
$\tan\left(\frac{\pi}{2}-x\right)=\cot x$	$\cot\left(\frac{\pi}{2}-x\right)=\tan x$	$\tan(90^\circ - x)=\cot x$	$\cot(90^\circ - x)=\tan x$
$\sec\left(\frac{\pi}{2}-x\right)=\csc x$	$\csc\left(\frac{\pi}{2}-x\right)=\sec x$	$\sec(90^\circ - x)=\csc x$	$\csc(90^\circ - x)=\sec x$

Remember, complementary angles add up to 90°, or in radians, add up to π/2.

Train Your Brain:

Working through the steps in the lesson examples and figuring out what is done in each step will help train your brain to see possibilities for putting together the puzzles of new problems!

YES, you may need to try more than one strategy to find a way to make it work!!

It is like doing a puzzle or a maze . . .

Be patient with the process and take your time!

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day & time.

Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!