Representing Vectors Key Concepts

Representing Vectors Lesson

Component Form

The component form of a vector with initial point $P(p_1, p_2)$ and terminal point $Q(q_1, q_2)$ is given by $\mathbf{v} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle$ where v_1 is the horizontal component and v_2 is the vertical component.

The magnitude (or length) of **v** is given by
$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{(v_1)^2 + (v_2)^2}$.

Equal Vectors

Two vectors are equal if the two vectors have the same magnitude and direction. Let $\mathbf{v} = \langle a, b \rangle$ and $\mathbf{w} = \langle c, d \rangle$; **v** = **w** if and only if *a* = *c* and *b* = *d*.

Operations with Vectors Lesson

Vector Addition and Subtraction

If $\mathbf{v} = \langle a, b \rangle$ and $\mathbf{w} = \langle c, d \rangle$, then $\mathbf{v} \pm \mathbf{w} = \langle a, b \rangle \pm \langle c, d \rangle = \langle a \pm c, b \pm d \rangle$.

Scalar Multiplication

If $\mathbf{v} = \langle a, b \rangle$ and *k* is a real number, then $k\mathbf{v} = k\langle a, b \rangle = \langle ka, kb \rangle$ with a magnitude of $|k|\|\mathbf{v}\|$.

If $k > 0$, then the direction of *k***v** is the same as the direction of **v**, and if $k < 0$, then the direction of *k***v** is the opposite of the direction of **v**.

Unit Vectors Lesson

Writing Vectors as a Linear Combination of i and j

If vector $\mathbf{v} = \langle v_1, v_2 \rangle$, then $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$ where v_1 and v_2 are the horizontal and vertical components of **v**, respectively.

If vector **v** has initial point $P = (x_1,\ y_1)$ and terminal point $Q = (x_2,\ y_2)$, then $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$.

Operations with Vectors in Terms of i and j

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$, $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, and *k* is a real number, then

$\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$,

$\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}$, and

$k\mathbf{v} = (ka_1)\mathbf{i} + (kb_1)\mathbf{j}$.

Finding a Unit Vector

The vector **u** is a scalar multiple of the vector **v**. The vector **u** has a magnitude of 1 and the same direction as **v**. The vector **u** is called a unit vector in the direction of **v**.

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

Direction Angle Lesson

Find the Direction Angle

Let θ be the direction angle and α be the reference angle. Find θ by finding the tangent of the reference angle, α.

$$\tan\alpha = \left|\frac{v_2}{v_1}\right|$$

When θ lies in quadrant I, $\theta = \alpha$.

When θ lies in quadrant II, $\theta = 180 - \alpha$.

When θ lies in quadrant III, $\theta = 180 + \alpha$.

When θ lies in quadrant IV, $\theta = 360 - \alpha$.

Writing Vectors in Terms of Magnitude and Direction

A vector **v** = a**i** + b**j** or $\langle a, b\rangle$ may be represented by its magnitude and direction as

$\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$ or

$\mathbf{v} = \langle \|\mathbf{v}\|\cos\theta, \|\mathbf{v}\|\sin\theta\rangle$,

where θ is the direction angle that measures the direction of the vector from the positive x-axis to **v**.

Dot Product Lesson

Dot Product

If $\mathbf{u} = \langle a_1, b_1\rangle$ and $\mathbf{v} = \langle a_2, b_2\rangle$, then the dot product, also called a scalar product, is denoted $\mathbf{u} \bullet \mathbf{v}$.

$\mathbf{u} \bullet \mathbf{v} = a_1 a_2 + b_1 b_2$

The product $\mathbf{u} \bullet \mathbf{v}$ is a scalar, or real number. It is not a vector.

Angle Between Two Vectors Lesson

The Angle Between Two Vectors

Given two vectors, **u** and **v**, and θ as the angle between vectors **u** and **v,** then $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta$.

Therefore, to find the angle θ between two vectors **u** and **v**,

$\cos\theta = \dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$, so

$\theta = \cos^{-1}\left(\dfrac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)$

where the angle θ is such that $0 \le \theta \le \pi$ or $0° \le \theta \le 180°$.

Orthogonal Vectors

Two vectors are orthogonal if the angle between them, θ, is equal to 90°. Since $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta$ and cos 90° = 0, then $\mathbf{u} \bullet \mathbf{v} = 0$. Therefore, if the dot product of two vectors is zero, then the two vectors are orthogonal.

Work

The work done, *W*, on a force, **F**, that moves an object from initial point *P* to terminal point *Q* can be calculated using the following formula:

$$W = \mathbf{F} \cdot \overrightarrow{PQ} = \|\mathbf{F}\| \left\|\overrightarrow{PQ}\right\| \cos\theta$$