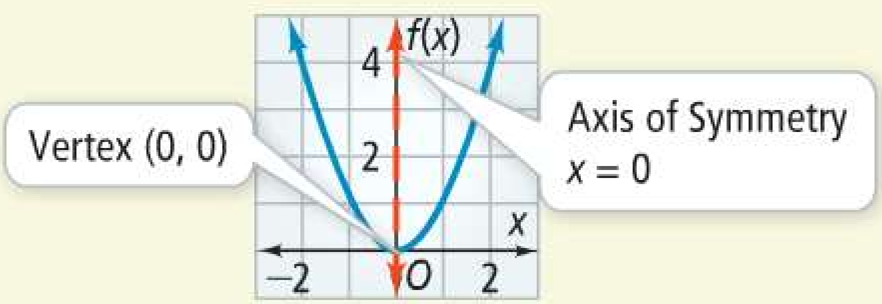
Chapter 4 Quadratic Functions and Equations

Quadratic Functions

Parent	$y = x^2$
Reflection across x-axis	$y = -x^2$
Stretch $(a > 1)$ Shrink $(0 < a < 1)$	$y = ax^2$
Translation horizontal by h vertical by k	$y = (x - h)^2 + k$
Vertex Form	$y = a(x - h)^2 + k$
Standard Form	$y = f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ and down if $a < 0$.

You can write every **quadratic function** in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. A **parabola** is the graph of a quadratic function. Every parabola has a vertex and an axis of symmetry. Shown below is the graph of the quadratic parent function $f(x) = x^2$.



The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$. The vertex of the parabola formed by a quadratic function is (h, k).

If $a > 0$, k is the **minimum value** of the function.

If $a < 0$, k is the **maximum value** of the function. The axis of symmetry is given by $x = h$.

The **standard form** of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$. When $a > 0$, the parabola opens up. When $a < 0$, the parabola opens down.

The axis of symmetry is the line $x = -\frac{b}{2a}$. The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, and the y-intercept is $(0, c)$.

The **zeros** of a quadratic function are the solutions of the related quadratic equation. You can find the zeros from a table or from the x-intercepts of the parabola that is the graph of the function. You can also find them by **factoring** the **standard form of a quadratic equation**, $ax^2 + bx + c = 0$, and using the **Zero-Product Property**.

To factor an expression of the form $ax^2 + bx + c$, when $a \neq 1$, you find numbers that have the product ac and sum b.

Factoring Perfect-Square Trinomials

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Factoring a Difference of Two Squares

$a^2 - b^2 = (a + b)(a - b)$

Zero-Product Property

If $ab = 0$, then $a = 0$ or $b = 0$.

You can solve a quadratic equation in the form $ax^2 + bx + c = 0$ by using the **Quadratic Formula**,

The Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant

The discriminant of a quadratic equation in the form $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

$b^2 - 4ac > 0 \Rightarrow$ two real solutions
$b^2 - 4ac = 0 \Rightarrow$ one real solution
$b^2 - 4ac < 0 \Rightarrow$ two complex solutions

Square Root of a Negative Real Number

For any positive number a,
$\sqrt{-a} = \sqrt{-1 \cdot a} = \sqrt{-1} \cdot \sqrt{a} = i\sqrt{a}$.
Example: $\sqrt{-5} = i\sqrt{5}$
Note that
$(\sqrt{-5})^2 = (i\sqrt{5})^2 = i^2(\sqrt{5})^2 = -1 \cdot 5 = -5$ (not 5).

A **complex number** is written in the form $a + bi$, where a and b are real numbers, and i is equal to $\sqrt{-1}$.

A system of quadratic equations can be solved by substitution or by graphing. You can use these methods to solve a linear–quadratic system or a quadratic–quadratic system. Use graphing to solve a quadratic system of inequalities.