Chapter 5 Polynomials and Polynomial Functions

End Behavior of a Polynomial Function

The end behavior of a polynomial function of degree n with leading term ax^n:

a	n	end behavior
positive	even	up and up
positive	odd	down and up
negative	even	down and down
negative	odd	up and down

A **polynomial function** is classified by degree. Its degree is the highest degree among its monomial term(s). The degree determines the possible number of **turning points** in the graph and the **end behavior** of the graph.

A turning point is a **relative maximum** or **relative minimum** of a polynomial function.

Factor Theorem

The expression $x - a$ is a linear factor of a polynomial if and only if the value a is a zero of the related polynomial function.

You can divide a polynomial by one of its factors to find another factor. When you divide by a linear factor, you can simplify this division by writing only the coefficients of each term. This is called **synthetic division**. The **Remainder Theorem** says that $P(a)$ is the remainder when you divide $P(x)$ by $x - a$.

Factoring a Sum or Difference of Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

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Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$.

Rational Root Theorem

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with integer coefficients.

Integer roots of $P(x) = 0$ must be factors of a_0.

Rational roots have reduced form $\frac{p}{q}$ where p is an integer factor of a_0 and q is an integer factor of a_n.

Conjugate Root Theorems

Suppose $P(x)$ is a polynomial with *rational* coefficients. If $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

Suppose $P(x)$ is a polynomial with *real* coefficients. If $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

Binomial Theorem

For every positive integer n, $(a + b)^n =$

$$P_0 a^n + P_1 a^{n-1} b + P_2 a^{n-2} b^2 + \cdots + P_{n-1} a b^{n-1} + P_n b^n$$

where $P_0, P_1, \ldots, P_n$ are the numbers in the nth row of Pascal's Triangle.

Rows 0–5 of Pascal's Triangle are shown below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

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