Name ______________________ Class ____________ Date ____________

8-4 Reteaching

Rational Expressions

Simplest form of a rational expression means the numerator and the denominator have no factors in common. You may have to restrict certain values of the variable(s) when you write in simplest form, because division by zero is undefined.

Problem

What is the expression $\frac{6x^3y^2 + 6x^2y^2 - 12xy^2}{3x^2y^3 - 12y^3}$ written in simplest form? State any restrictions on the variables.

$\frac{6xy^2(x^2 + x - 2)}{3y^3(x^2 - 4)}$ Factor $6xy^2$ out of the numerator and $3y^3$ out of the denominator.

$\frac{6xy^2(x + 2)(x - 1)}{3y^3(x + 2)(x - 2)}$ Factor $(x^2 + x - 2)$ and $(x^2 - 4)$.

$\frac{(2 \cdot \cancel{3} \cdot x \cdot \cancel{y} \cdot \cancel{y})\cancel{(x + 2)}(x - 1)}{(\cancel{3} \cdot y \cdot \cancel{y} \cdot \cancel{y})\cancel{(x + 2)}(x - 2)}$ Divide out the common factors.

$\frac{2x(x - 1)}{y(x - 2)}$ Write the remaining factors.

Look at the **original** expression.

$\frac{6xy^2(x + 2)(x - 1)}{3y^3(x + 2)(x - 2)}$ is undefined if

$3y^3 = 0$, $x + 2 = 0$, or $x - 2 = 0$.

So, $y \neq 0$, $x \neq -2$, and $x \neq 2$.

Look at the **simplified** expression.

$\frac{2x(x - 1)}{y(x - 2)}$ is undefined if

$y = 0$ or $x - 2 = 0$.

So, $y \neq 0$ and $x \neq 2$.

In simplest form, the expression is $\frac{2x(x - 1)}{y(x - 2)}$, where $y \neq 0$, $x \neq -2$, and $x \neq 2$.

Exercises

Simplify each rational expression. State any restrictions on the variable.

1. $\frac{x^2 + x}{x^2 + 2x}$ **$\frac{x + 1}{x + 2}$; $x \neq -2, 0$**

2. $\frac{x^2 - 5x}{x^2 - 25}$ **$\frac{x}{x + 5}$; $x \neq \pm 5$**

3. $\frac{x^2 + 3x - 18}{x^2 - 36}$ **$\frac{x - 3}{x - 6}$; $x \neq \pm 6$**

4. $\frac{4x^2 - 36}{x^2 + 10x + 21}$ **$\frac{4x - 12}{x + 7}$; $x \neq -3, -7$**

5. $\frac{3x^2 - 12}{x^2 - x - 6}$ **$\frac{3x - 6}{x - 3}$; $x \neq -2, 3$**

6. $\frac{x^2 - 9}{2x + 6}$ **$\frac{x - 3}{2}$; $x \neq -3$**

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8-4 Reteaching (continued)

Rational Expressions

To find the quotient $\frac{a}{b} \div \frac{c}{d}$, multiply by the reciprocal of the divisor. Invert the divisor and then multiply: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

To divide any rational expression by a second rational expression, follow this pattern.

Problem

What is the quotient $\frac{x^2 - 2x - 35}{2x^3 - 3x^2} \div \frac{7x - 49}{4x^3 - 9x}$, written in simplest form? State any restrictions on the variable.

Step 1 Invert the divisor, which is the second expression.

Reciprocal of $\frac{7x - 49}{4x^3 - 9x}$ is $\frac{4x^3 - 9x}{7x - 49}$

Step 2 Write the quotient as one rational expression.

$$\frac{x^2 - 2x - 35}{2x^3 - 3x^2} \div \frac{7x - 49}{4x^3 - 9x} = \frac{x^2 - 2x - 35}{2x^3 - 3x^2} \cdot \frac{4x^3 - 9x}{7x - 49} = \frac{(x^2 - 2x - 35)(4x^3 - 9x)}{(2x^3 - 3x^2)(7x - 49)}$$

Step 3 Factor each polynomial in the numerator and denominator.

$$\frac{(x^2 - 2x - 35)(4x^3 - 9x)}{(2x^3 - 3x^2)(7x - 49)} = \frac{[(x - 7)(x + 5)][x(2x + 3)(2x - 3)]}{[x^2(2x - 3)][7(x - 7)]}$$

Step 4 Divide out the common factors and simplify.

$$\frac{[\cancel{(x - 7)}(x + 5)][\cancel{x}(2x + 3)\cancel{(2x - 3)}]}{[(x \cdot \cancel{x})\cancel{(2x - 3)}][7\cancel{(x - 7)}]} = \frac{(x + 5)(2x + 3)}{7x} = \frac{2x^2 + 13x + 15}{7x}$$

Therefore, $\frac{x^2 - 2x - 35}{2x^3 - 3x^2} \div \frac{7x - 49}{4x^3 - 9x} = \frac{2x^2 + 13x + 15}{7x}$, where $x \neq 0, \pm\frac{3}{2}, 7$.

Exercises

Divide. State any restrictions on the variables.

7. $\frac{3x + 12}{2x - 8} \div \frac{x^2 + 8x + 16}{x^2 - 8x + 16}$ **$\frac{3x - 12}{2x + 8}$; $x \neq \pm 4$**

8. $\frac{2x^2 - 16x}{x^2 - 9x + 8} \div \frac{2x}{5x - 5}$ **5; $x \neq 0, 1, 8$**

9. $\frac{2x - 10}{3x - 21} \div \frac{x - 5}{4x - 28}$ **$\frac{8}{3}$; $x \neq 5, 7$**

10. $\frac{4x - 16}{4x} \div \frac{x^2 - 2x - 8}{3x + 6}$ **$\frac{3}{x}$; $x \neq -2, 0, 4$**