Name _____ Class _____ Date _____

8-5 Reteaching

Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is a lot like adding and subtracting fractions. Before you can add or subtract the expressions, they must have a common denominator. The easiest common denominator to work with is the *least common denominator*, or LCD.

Problem

What is the LCD of $\frac{6x}{x^3 + 2x^2}$ and $\frac{5}{x^3 + x^2 - 2x}$?

$x^3 + 2x^2 = x^2(x + 2)$
$x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x + 2)(x - 1)$ — Completely factor each denominator.

$x^2, (x + 2), x, (x + 2), (x - 1)$ — Make a list of all the factors.

$x^2, (x + 2), x, \cancel{(x + 2)}, (x - 1)$ — Cross off any repeated factors.

$x^2, (x + 2), \cancel{x}, \cancel{(x + 2)}, (x - 1)$ — When the only difference between factors is the exponent (like x^2 and x), cross off all but the factor with the greatest exponent.

$x^2(x + 2)(x - 1)$ — Multiply the remaining factors on the list. The product is the LCD.

The LCD of $\frac{6x}{x^3 + 2x^2}$ and $\frac{5}{x^3 + x^2 - 2x}$ is $x^2(x + 2)(x - 1)$.

Exercises

Assume that the polynomials given are the denominators of rational expressions. Find the LCD of each set.

1. $x + 3$ and $2x + 6$ $2(x + 3)$
2. $2x - 1$ and $3x + 4$ $(2x - 1)(3x + 4)$
3. $x^2 - 4$ and $x + 2$ $(x + 2)(x - 2)$
4. $x^2 + 7x + 12$ and $x + 4$ $(x + 3)(x + 4)$
5. $x^2 + 5$ and $x - 25$ $(x^2 + 5)(x - 25)$
6. x^3 and $6x^2$ $6x^3$
7. x, $2x$, and $4x^3$ $4x^3$
8. $x^2 + 8x + 16$ and $x + 4$ $(x + 4)^2$
9. $x^2 + 4x - 5$ and $x^3 - x^2$
   $x^2(x - 1)(x + 5)$
10. $x^2 - 9$ and $x^2 + 2x - 3$
    $(x + 3)(x - 3)(x - 1)$

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8-5 Reteaching (continued)

Adding and Subtracting Rational Expressions

To find the sum or difference of rational expressions with unlike denominators:

- completely factor each denominator
- identify the least common denominator, or LCD
- multiply each expression by the factors needed to produce the LCD
- add or subtract numerators, and put the result over the LCD

Problem

What is the difference of $\frac{2x}{3x^2 + 5x} - \frac{14}{3x^2 + 26x + 35}$ in simplest form? State any restrictions on the variable.

$3x^2 + 5x = x(3x + 5)$
$3x^2 + 26x + 35 = (3x + 5)(x + 7)$ — Completely factor each denominator.

$x(3x + 5)(x + 7)$ — Identify the LCD.

$\left[\frac{2x}{x(3x + 5)} \cdot \frac{(x + 7)}{(x + 7)}\right] - \left[\frac{14}{(3x + 5)(x + 7)} \cdot \frac{x}{x}\right]$ — Multiply to produce the LCD.

$= \frac{2x(x + 7)}{x(3x + 5)(x + 7)} - \frac{14x}{x(3x + 5)(x + 7)}$

$= \frac{2x(x + 7) - 14x}{x(3x + 5)(x + 7)}$ — Subtract the numerators.

$= \frac{2x^2 + 14x - 14x}{x(3x + 5)(x + 7)}$ — Distribute.

$= \frac{2x}{3x^2 + 26x + 35}$ — Simplify.

Therefore, $\frac{2x}{3x^2 + 5x} - \frac{14}{3x^2 + 26x + 35} = \frac{2x}{3x^2 + 26x + 35}$, where $x \neq -7, -\frac{5}{3}, 0$.

Exercises

Simplify each sum or difference. State any restrictions on the variable.

11. $\frac{y}{y - 1} + \frac{2}{1 - y}$ $\frac{y - 2}{y - 1}$; $y \neq 1$

12. $\frac{3}{x + 2} + \frac{2}{x^2 - 4}$ $\frac{3x - 4}{(x + 2)(x - 2)}$; $x \neq \pm 2$

13. $\frac{x}{x^2 + 5x + 6} - \frac{2}{x^2 + 3x + 2}$
$\frac{x - 3}{(x + 1)(x + 3)}$; $x \neq -1, -2, -3$

14. $\frac{4x + 1}{x^2 - 4} - \frac{3}{x - 2}$
$\frac{x - 5}{(x + 2)(x - 2)}$; $x \neq \pm 2$