Name _____ Class _____ Date _____

9-3 Reteaching

Geometric Sequences

- A geometric sequence has a constant ratio between consecutive terms. This number is called the common ratio.
- A geometric sequence can be described by a recursive formula, $a_n = a_{n-1} \cdot r$, or as an explicit formula, $a_n = a \cdot r^{n-1}$.

Problem

Find the 12th term of the geometric sequence 5, 15, 45, ...

5, 15, 45, . . .

$r = \frac{15}{5} = \frac{45}{15} = 3$ Find r by calculating the common ratio between consecutive terms. This is a geometric sequence because there is a common ratio between consecutive terms.

$a_n = 5(3)^{n-1}$ Substitute $a = 5$ and $r = 3$ into the explicit formula to find a formula for the nth term of the sequence.

$a_{12} = 5(3)^{11}$ Substitute $n = 12$ to find the 12th term of the sequence.

$a_{12} = 885{,}735$ Remember to first calculate 3^{11}, then multiply by 5.

Exercises

Find the indicated term of the geometric sequence.

1. 4, 2, 1, . . . Find a_{10}. $\frac{1}{128}$

2. 5, $\frac{15}{2}$, $\frac{45}{4}$, . . . Find a_8. $\frac{10{,}935}{128}$

3. 6, −2, $\frac{2}{3}$, . . . Find a_{12}. $-\frac{2}{59{,}049}$

4. 1, $-\frac{2}{3}$, $\frac{4}{9}$, . . . Find a_7. $\frac{64}{729}$

5. 100, 200, 400, . . . Find a_9. 25,600

6. 8, 32, 128, . . . Find a_4. 512

Write the explicit formula for each sequence. Then generate the first five terms.

7. $a_1 = 1, r = \frac{1}{2}$
$a_n = 1\left(\frac{1}{2}\right)^{n-1}$; 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$

8. $a_1 = 2, r = 3$
$a_n = 2(3)^{n-1}$; 2, 6, 18, 54, 162

9. $a_1 = 12, r = 3$
$a_n = 12(3)^{n-1}$; 12, 36, 108, 324, 972

10. $a_1 = 1, r = \frac{1}{4}$
$a_n = 1\left(\frac{1}{4}\right)^{n-1}$; 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, $\frac{1}{256}$

11. $a_1 = 5, r = \frac{1}{10}$
$a_n = 5\left(\frac{1}{10}\right)^{n-1}$; 5, $\frac{1}{2}$, $\frac{1}{20}$, $\frac{1}{200}$, $\frac{1}{2000}$

12. $a_1 = 1, r = \frac{1}{3}$
$a_n = 1\left(\frac{1}{3}\right)^{n-1}$; 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$

13. $a_1 = 5, r = 2$
$a_n = 5(2)^{n-1}$; 5, 10, 20, 40, 80

14. $a_1 = 1, r = 3$
$a_n = 1(3)^{n-1}$; 1, 3, 9, 27, 81

15. $a_1 = 3, r = 6$
$a_n = 3(6)^{n-1}$; 3, 18, 108, 648, 3888

16. $a_1 = 3, r = 3$
$a_n = 3(3)^{n-1}$; 3, 9, 27, 81, 243

17. $a_1 = 2, r = 2$
$a_n = 2(2)^{n-1}$; 2, 4, 8, 16, 32

18. $a_1 = 2, r = \frac{1}{2}$
$a_n = 2\left(\frac{1}{2}\right)^{n-1}$; 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$

19. $a_1 = 1, r = \frac{1}{5}$
$a_n = 1\left(\frac{1}{5}\right)^{n-1}$; 1, $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$, $\frac{1}{625}$

20. $a_1 = 3, r = 4$
$a_n = 3(4)^{n-1}$; 3, 12, 48, 192, 768

21. $a_1 = 5, r = \frac{1}{4}$
$a_n = 5\left(\frac{1}{4}\right)^{n-1}$; 5, $\frac{5}{4}$, $\frac{5}{16}$, $\frac{5}{64}$, $\frac{5}{256}$

9-3 Reteaching (continued)

Geometric Sequences

Problem

From 2000 to 2009, your friend's landlord has been allowed to raise her rent by the same percent each year. In 2000, her rent was \$1000, and in 2003, her rent was \$1092.73. What was her rent in 2009?

Step 1 Identify key information in the problem.
You know that your friend's rent was \$1000 in 2000. This means $a = 1000$. You also know that her rent in 2003 was \$1092.73. This means that $a_4 = 1092.73$. Her rent is raised by the same percent each year, which is the same as multiplying by a constant (e.g., *a* 5% increase is the same as multiplying by 1.05).

Step 2 Identify missing information.
You need to find the common ratio *r* in order to find the rent in 2009, a_{10}.

Step 3 Use the explicit formula to find *r*.

$a_n = ar^{n-1}$		Write the explicit formula.
$1092.73 = (1000)r^{4-1}$		Substitute $a = 1000$, $a_4 = 1092.73$, and $n = 4$.
$1092.73 = 1000r^3$		Simplify.
$1.09273 = r^3$		Divide each side by 1000.
$1.03 = r$		Take the cube root of both sides.

Step 4 Use the value of r to find the rent in 2009, a_{10}.

$a_n = ar^{n-1}$		Write the explicit formula.
$a_{10} = (1000)(1.03)^{10-1}$		Substitute $a = 1000$, $r = 1.03$, and $n = 10$.
$a_{10} = (1000)(1.03)^9$		Simplify.
$a_{10} \approx 1304.77$		Compute. Round to the nearest hundredth.

Your friend's rent was \$1304.77 in 2009.

Exercises

22. An athlete is training for a bicycle race. She increases the amount she bikes by the same percent each day. If she bikes 10 mi on the first day, and 12.1 mi on the third day, how much will she bike on the fifth day? By what percent does she increase the amount she bikes each day? **14.641 mi; 10%**

23. By clipping coupons and eating more meals at home, your family plans to decrease their monthly food budget by the same percent each month. If they budgeted \$600 in January and \$514.43 in April, how much will they budget in December? **\$341.28**

24. From 2005 to 2009, a teen raised her babysitting rates by a fixed percent every year. If she charged \$8/h in 2005 and \$10.04/h in 2007, how much did she charge in 2009? What is her percent of increase each year? **\$12.59/h; 12%**