

9-4 Reteaching

Arithmetic Series

Summation notation shows the upper limit, lower limit, and explicit formula for the terms of a series.

To find the sum of an arithmetic series written in summation notation:

- list the terms and add them, or use the formula $S_n = \frac{n}{2}(a_1 + a_n)$

Problem

What is the sum of the series written in summation notation?

a. $\sum_{n=2}^4 (5 - 2n)$

$$\sum_{n=2}^4 (5 - 2n)$$

$n = 2$ $n = 3$ $n = 4$
 $5 - 2(2)$ $5 - 2(3)$ $5 - 2(4)$

Circle the upper and lower limits. Box the explicit formula.

In circles, write all possible values of n , from the lower limit to the upper limit.

Under each circle copy the explicit formula, substituting the value in the circle for n .

$$\begin{aligned} \sum_{n=2}^4 (5 - 2n) &= 5 - 2(2) + 5 - 2(3) + 5 - 2(4) \\ &= 1 + (-1) + (-3) \\ &= -3 \end{aligned}$$

The value of the series is the sum of the values in the boxes.

Evaluate each expression.

Find the sum of the terms.

The sum of the series is -3 .

b. $\sum_{n=1}^{15} (4n - 1)$

Use the formula $S_n = \frac{n}{2}(a_1 + a_n)$. First, find n , a_1 , and a_n . The upper limit is 15.

$$a_1 = 4(1) - 1 = 3 \quad \text{Evaluate the explicit formula at } n = 1.$$

$$a_n = a_{15} = 4(15) - 1 = 59 \quad \text{Evaluate the explicit formula at } n = 15.$$

$$S_n = \frac{15}{2}(3 + 59) \quad \text{Substitute } n = 15, a_1 = 3, \text{ and } a_n = 59.$$

$$= 465 \quad \text{Simplify.}$$

The sum of the series is 465.

Exercises

Find the sum of each finite series.

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|---------------------------------------|---|--------------------------------------|--|
| 1. $\sum_{n=1}^3 (n - 4)$ -6 | 2. $\sum_{n=1}^4 \frac{1}{3}n$ $\frac{10}{3}$ | 3. $\sum_{n=3}^8 (3n - 1)$ 93 | 4. $\sum_{n=3}^8 \frac{2n}{3}$ 22 |
| 5. $\sum_{n=3}^9 (4 - 2n)$ -56 | 6. $\sum_{n=1}^5 8n$ 120 | 7. $\sum_{n=2}^7 4n$ 108 | 8. $\sum_{n=1}^7 (3 - 2n)$ -35 |

9-4 **Reteaching** (continued)

Arithmetic Series

Problem

The debate club is offering a prize at the end of 10 weeks to a current member who brings three new members for the first meeting, and then increases the number of new members they bring each week by two thereafter. One member qualified for the prize with the minimum number of new members. How many new members did the member bring at Week 10? For all 10 weeks?

Step 1 Identify key information in the problem.

To win the prize, a member must bring three members to the first meeting, so $a = 3$.

A member must also bring two more new members to each meeting, so $d = 2$.

The contest extends for 10 weeks, so $n = 10$.

Step 2 Identify the information you are trying to find.

You want to find the 10th term, a_{10} , and the sum of the first 10 terms, S_{10} .

Step 3 Use the explicit formula to find a_{10} .

$$\begin{aligned} a_n &= a + (n - 1)d && \text{Write the explicit formula.} \\ a_{10} &= 3 + (10 - 1)2 && \text{Substitute } a = 3, d = 2, \text{ and } n = 10. \\ a_{10} &= 21 && \text{Simplify.} \end{aligned}$$

To win the prize, a member brought 21 new members to a meeting at Week 10.

Step 4 Use the value of a_{10} to find the total number of new members brought by the winner.

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) && \text{Write the formula for the sum of an arithmetic series.} \\ S_{10} &= \frac{10}{2}(3 + 21) && \text{Substitute } a_1 = 3, a_{10} = 21, \text{ and } n = 10. \\ S_{10} &= 120 && \text{Simplify.} \end{aligned}$$

The debate club had 120 new members brought in by the winner of the contest.

Exercises

9. The seating arrangement for a recital uses 20 seats in the first row and two additional seats in each row thereafter. How many seats will be in the eighth row? In the ninth row? How many seats total are there in the first nine rows?

34 seats; 36 seats; 252 seats

10. With the help of a tutor, a student's weekly quiz scores have increased during the first four quizzes: 65, 70, 75, and 80. If the scores continue to increase at this rate, what will be the score in the 7th week? In the 8th week? What is the total of the first eight scores?

95; 100; 660