Name _____ Class _____ Date _____

11-9 Reteaching

Binomial Distributions

Suppose you repeat an experiment n times, and each time you run the experiment it has a probability of success p and a probability of failure q. Then, the probability of x successes in n trials is:

$$ {}_nC_x p^x q^{n-x}, \text{ where } q = 1 - p $$

Problem

What is the probability of two successes in five trials, where the probability of success for each trial is 0.2?

$$
\begin{aligned}
{}_nC_x &= {}_5C_2 && \text{Find } {}_nC_x. \\
&= \frac{5!}{2!(5-2)!} \\
&= 10 \\
q &= 1 - p && \text{Find } q. \\
&= 1 - .2 \\
&= 0.8 \\
P(2 \text{ successes}) &= {}_5C_2 (0.2)^2 (0.8)^{5-2} && \text{Substitute for } n, x, p, \text{ and } q \text{ in the formula.} \\
&= 10(0.2)^2(0.8)^3 && \text{Simplify.} \\
&= 10(0.04)(0.512) \\
&= 0.2048
\end{aligned}
$$

The probability of two successes in five trials is about 20%.

Exercises

Find the probability of x successes in n trials for the given probability of success p on each trial. Round to the nearest tenth of a percent.

1. $x = 3, n = 4, p = 0.3$ **7.6%**
2. $x = 4, n = 6, p = 0.1$ **0.1%**
3. $x = 7, n = 9, p = 0.4$ **2.1%**
4. $x = 5, n = 6, p = 0.3$ **1.0%**
5. A light fixture contains six light bulbs. With normal use, each bulb has a 95% chance of lasting for 2 yr. What is the probability that all six bulbs last for 2 yr? **about 73.5%**
6. Use the information from Exercise 5. What is the probability that five of the six bulbs will last for 2 yr? **about 23.2%**
7. Suppose the bulbs have an 80% chance of lasting for 2 yr. Find the probability that three of the six bulbs will last for 2 yr. **8.2%**

11-9 Reteaching (continued)

Binomial Distributions

Suppose you repeat an experiment n times, and each time you run the experiment it has a probability of success p and a probability of failure q. You can expand the binomial $(p + q)^n$ to find the distribution of binomial probabilities.

- The first term of the expansion is the probability of n successes out of n trials.
- The second term of the expansion is the probability of $n - 1$ successes out of n trials.
- The third term of the expansion is the probability of $n - 2$ successes out of n trials.
- The pattern continues to the last term, the probability of zero successes out of n trials.

Problem

Find the distribution of binomial probabilities for four trials with a probability of success of 0.3 for each trial. What is the probability of at least three successes?

Step 1 Use the Binomial Theorem to expand $(p + q)^n$.

$${}_4C_4(0.3)^4(0.7)^{4-4} + {}_4C_3(0.3)^3(0.7)^{4-3} + {}_4C_2(0.3)^2(0.7)^{4-2} + {}_4C_1(0.3)^1(0.7)^{4-1} + {}_4C_0(0.3)^0(0.7)^{4-0}$$

Step 2 Simplify.

$$= (0.3)^4 + 4(0.3)^3(0.7)^1 + 6(0.3)^2(0.7)^2 + 4(0.3)^1(0.7)^3 + (0.7)^4$$
$$= 0.0081 + 0.0756 + 0.2646 + 0.4116 + 0.2401$$

Step 3 Determine the distribution of binomial probabilities.

$P(4 \text{ successes}) = 0.0081$ $P(3 \text{ successes}) = 0.0756$ $P(2 \text{ successes}) = 0.2646$
$P(1 \text{ success}) = 0.4116$ $P(0 \text{ successes}) = 0.2401$

Step 4 Find the probability of at least 3 successes.

$$P(\text{at least 3 successes}) = P(4 \text{ successes}) + P(3 \text{ successes})$$
$$= 0.0081 + 0.0756 = 0.0837$$

The probability of at least three successes in four trials is 8.37%.

Exercises

8. In a population of laboratory mice, the probability that a mouse has black spots is 0.85. What is the probability of randomly choosing 7 mice and getting at least 5 mice with black spots? about 92.6%

9. A construction site has a 97% rate of accident-free workdays. What is the probability of no more than 1 accident in the next 5 days? about 99.2%