

13-3 Reteaching

Radian Measure

- A central angle that measures π radians intercepts an arc that forms a semicircle. It is a 180° rotation from the initial side to the terminal side of the angle.
- When converting radians to degrees or degrees to radians, use the proportion $\frac{\text{degree measure}}{360} = \frac{\text{radian measure}}{2\pi}$.

Problem

What is the radian measure of an angle of 225° ?

$$\frac{225}{360} = \frac{x}{2\pi} \quad \text{Substitute 225 for degree measure and a variable for radian measure.}$$

$$360x = 450\pi \quad \text{Cross multiply.}$$

$$x = \frac{450\pi}{360} \quad \text{Divide both sides by 360.}$$

$$x = \frac{5\pi}{4} \quad \text{Simplify.}$$

$$x \approx 3.93 \quad \text{Use a calculator.}$$

Check $\frac{\theta}{360} = \frac{\frac{5}{4}\pi}{2\pi}$ Check by substituting the radians into the proportion and solving for degrees.

$$\frac{\theta}{360} = \frac{\frac{5}{4}\cancel{\pi}}{2\cancel{\pi}} \quad \text{Cancel } \pi \text{ since it is in the numerator and denominator.}$$

$$2\theta = 450 \quad \text{Cross multiply.}$$

$$\theta = 225 \quad \text{Divide both sides by 2. This gives the degree measure.}$$

An angle of 225° measures about 3.93 radians.

Exercises

Write each measure in radians and check.

1. $20^\circ \approx \frac{\pi}{9} \approx 0.35$

2. $150^\circ \approx \frac{5\pi}{6} \approx 2.62$

3. $45^\circ \approx \frac{\pi}{4} \approx 0.79$

4. $-110^\circ \approx -\frac{11\pi}{18} \approx -1.92$

5. $315^\circ \approx \frac{7\pi}{4} \approx 5.50$

6. $320^\circ \approx \frac{16\pi}{9} \approx 5.59$

Write each measure in degrees and check.

7. $-\frac{3\pi}{2} \approx -270^\circ$

8. $\frac{5\pi}{3} \approx 300^\circ$

9. $\frac{\pi}{12} \approx 15^\circ$

10. $\frac{8\pi}{5} \approx 288^\circ$

11. $-\frac{7\pi}{6} \approx -210^\circ$

12. $\frac{9\pi}{2} \approx 810^\circ$

13-3 Reteaching (continued)

Radian Measure

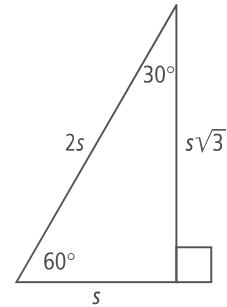
You can use Special Right Triangles to find the exact values for the cosine and sine of radian measures.

Problem

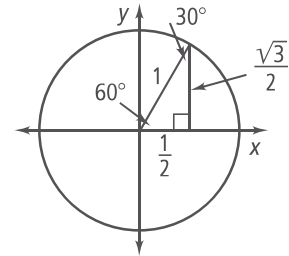
What are the exact values of $\cos\left(\frac{\pi}{3}\text{ radians}\right)$ and $\sin\left(\frac{\pi}{3}\text{ radians}\right)$?

Step 1 Find the angle measure in degrees. $\frac{\pi}{3} = \frac{180^\circ}{3} = 60^\circ$

Step 2 Recall the 30° - 60° - 90° triangle from Geometry. In the unit circle, the hypotenuse is 1. So, $2s = 1$, or $s = \frac{1}{2}$. Therefore the side opposite the 30° angle is $\frac{1}{2}$ and the side opposite the 60° angle is $\frac{\sqrt{3}}{2}$.



Step 3 Draw the angle on the unit circle. Complete a 30° - 60° - 90° triangle. Label the sides of the triangle.



Step 4 The cosine is the x -coordinate of the point at which the terminal side of the angle intersects the unit circle. The sine is the y -coordinate.
 $\cos\left(\frac{\pi}{3}\text{ radians}\right) = \frac{1}{2}$ and $\sin\left(\frac{\pi}{3}\text{ radians}\right) = \frac{\sqrt{3}}{2}$.

Exercises

The measure θ of an angle in standard position is given. Find the exact values of $\cos \theta$ and $\sin \theta$ for each angle measure.

13. $-\frac{\pi}{6}$ radians
 $\frac{\sqrt{3}}{2}, -\frac{1}{2}$

14. $\frac{3\pi}{4}$ radians
 $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

15. $-\frac{\pi}{3}$ radians
 $\frac{1}{2}, -\frac{\sqrt{3}}{2}$

16. $-\frac{2\pi}{3}$ radians
 $-\frac{1}{2}, -\frac{\sqrt{3}}{2}$

17. $\frac{5\pi}{6}$ radians
 $-\frac{\sqrt{3}}{2}, \frac{1}{2}$

18. $\frac{7\pi}{4}$ radians
 $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$