

13-6 Reteaching

The Tangent Function

Like the sine and cosine functions, the standard form of a tangent function is $y = a \tan b\theta$, where $a \neq 0$, $b > 0$, and θ is measured in radians. However, the graph of a tangent function is different in several important ways.

- The amplitude is undefined.
- The period is $\frac{\pi}{b}$.
- One cycle occurs between vertical asymptotes at $\theta = -\frac{\pi}{2b}$ and $\theta = \frac{\pi}{2b}$.
- This cycle also passes through $(-\frac{\pi}{4b}, -a)$ and $(\frac{\pi}{4b}, a)$.
- Vertical asymptotes occur at the end of each cycle.

Problem

What are the period and asymptotes of the graph of $y = -2 \tan(3\theta)$ in the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$? Find two points on the graph that are not on the x -axis.

$$a = -2, b = 3$$

Compare $y = -2 \tan(3\theta)$ to $y = a \tan b\theta$.

$$\frac{\pi}{b} = \frac{\pi}{3}$$

Calculate the period.

$$\theta = -\frac{\pi}{2b} = -\frac{\pi}{2(3)} = -\frac{\pi}{6}$$

Find one asymptote.

$$\theta = \frac{\pi}{2b} = \frac{\pi}{2(3)} = \frac{\pi}{6}$$

Find another asymptote.

$$\left(-\frac{\pi}{4b}, -a\right) = \left(-\frac{\pi}{4(3)}, -(-2)\right) = \left(-\frac{\pi}{12}, 2\right)$$

Find one point on the graph.

$$\left(\frac{\pi}{4b}, a\right) = \left(\frac{\pi}{4(3)}, -2\right) = \left(\frac{\pi}{12}, -2\right)$$

Find another point on the graph.

Exercises

Find the period and two asymptotes of the graph of each tangent function. Then find two points on each graph that are not on the x -axis.

1. $y = 4 \tan \theta$

Sample: $\pi; \theta = -\frac{\pi}{2}, \theta = \frac{\pi}{2};$
 $\left(-\frac{\pi}{4}, -4\right), \left(\frac{\pi}{4}, 4\right)$

2. $y = -\tan 2\theta$

Sample: $\frac{\pi}{2}; \theta = -\frac{\pi}{4}, \theta = \frac{\pi}{4};$
 $\left(-\frac{\pi}{8}, 1\right), \left(\frac{\pi}{8}, -1\right)$

3. $y = \tan \frac{1}{2}\theta$

Sample: $2\pi; \theta = -\pi, \theta = \pi;$
 $\left(-\frac{\pi}{2}, -1\right), \left(\frac{\pi}{2}, 1\right)$

13-6 **Reteaching** (continued)

The Tangent Function

The tangent function is a discontinuous periodic function. Each cycle of the graph occurs between vertical asymptotes. To graph a tangent function:

- Find the period, asymptotes, and two points on one cycle of the graph.
- Graph this cycle.
- Find additional asymptotes by adding positive and negative multiples of the period to the first two asymptotes.
- Find additional points on the graph by adding positive and negative multiples of the period to the x -coordinates of the first two points you found.

Problem

Sketch four cycles of the graph of $y = -2 \tan(3\theta)$.

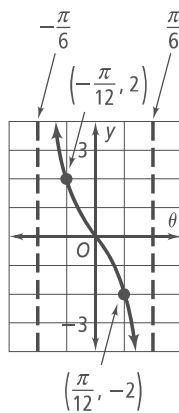
Step 1

Find the period, asymptotes, and two points on one cycle.

The period is $\frac{\pi}{3}$, asymptotes are at $\theta = -\frac{\pi}{6}$ and $\theta = \frac{\pi}{6}$, and two points on the graph are $(-\frac{\pi}{12}, 2)$ and $(\frac{\pi}{12}, -2)$.

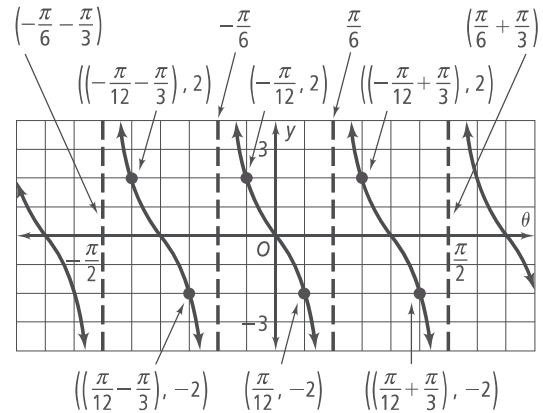
Step 2

Graph the first cycle.



Step 3

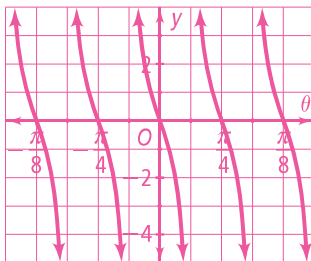
Graph additional cycles.



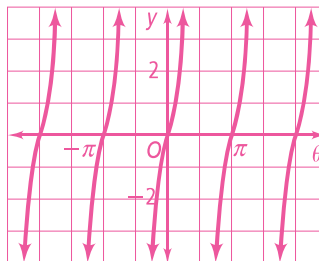
Exercises

Graph at least three cycles of each tangent function.

4. $y = -2 \tan 4\theta$



5. $y = 3 \tan \theta$



6. $y = \tan \theta$

