

13-8 Reteaching

Reciprocal Trigonometric Functions

You have already worked with sine, cosine, and tangent functions. The reciprocals of these functions are also trigonometric functions:

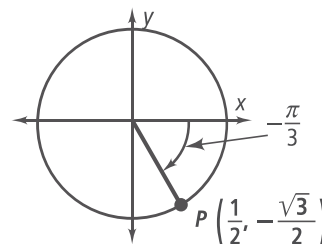
<u>Cosecant</u>	<u>Secant</u>	<u>Cotangent</u>
$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$
$\sin \theta = \frac{1}{\csc \theta}$	$\cos \theta = \frac{1}{\sec \theta}$	$\tan \theta = \frac{1}{\cot \theta}$

Problem

What is the exact value of $\sec\left(-\frac{\pi}{3}\right)$? Do not use a calculator.

Step 1 Find the reciprocal of $\sec\left(-\frac{\pi}{3}\right)$. $\frac{1}{\sec\left(-\frac{\pi}{3}\right)} = \cos\left(-\frac{\pi}{3}\right)$

Step 2 Draw the unit circle.
Draw the terminal side of the angle $-\frac{\pi}{3}$.



Step 3 Label the coordinates of the point where the unit circle intersects the terminal side of the angle $-\frac{\pi}{3}$.

Step 4 Find the exact value of $\cos\left(-\frac{\pi}{3}\right)$.
 $\cos\left(-\frac{\pi}{3}\right) = x\text{-coordinate of point } P = \frac{1}{2}$.

Step 5 Use the definition of secant. $\sec\left(-\frac{\pi}{3}\right) = \frac{1}{\cos\left(-\frac{\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$

Exercises

Find the exact value of each expression. Do not use a calculator.

- | | | | |
|--|---|---|---|
| 1. $\cot \frac{\pi}{6} \quad \sqrt{3}$ | 2. $\sec\left(-\frac{3\pi}{4}\right) \quad -\sqrt{2}$ | 3. $\csc\left(-\frac{\pi}{2}\right) \quad -1$ | 4. $\sec \frac{5\pi}{3} \quad 2$ |
| 5. $\csc \frac{\pi}{4} \quad \sqrt{2}$ | 6. $\cot \frac{2\pi}{3} \quad -\frac{\sqrt{3}}{3}$ | 7. $\sec(3\pi) \quad -1$ | 8. $\csc\left(-\frac{\pi}{6}\right) \quad -2$ |

13-8 **Reteaching** (continued)

Reciprocal Trigonometric Functions

The graphs of cosecant, secant, and cotangent functions are related to the graphs of sine, cosine, and tangent functions.

- The graph of a cosecant function has a vertical asymptote where the value of the related sine function is zero.
- The graph of a secant function has a vertical asymptote where the value of the related cosine function is zero.
- The graph of a cotangent function is a reflection across a vertical line of the related tangent function.

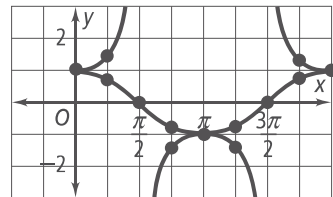
Problem

What are the graphs of $y = \cos x$ and $y = \sec x$ in the interval from 0 to 2π ?

Step 1 Make a table of values. Use the fact that $\sec \theta = \frac{1}{\cos \theta}$. The graph of $y = \sec x$ has asymptotes where $\cos x$ is equal to zero.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\cos \theta$	1	0.707	0	-0.707	-1	-0.707	0	0.707	1
$\frac{1}{\cos \theta}$	$\frac{1}{1}$	$\frac{1}{0.707}$	$\frac{1}{0}$	$\frac{1}{-0.707}$	$\frac{1}{-1}$	$\frac{1}{-0.707}$	$\frac{1}{0}$	$\frac{1}{0.707}$	$\frac{1}{1}$
$\sec \theta$	1	1.414	—	-1.414	-1	-1.414	—	1.414	1

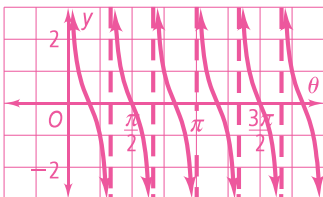
Step 2 Plot the points from the table. Connect the points with smooth curves.



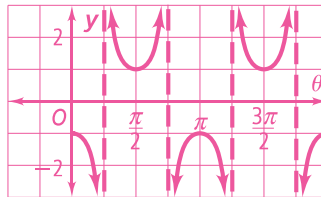
Exercises

Sketch each graph in the interval from 0 to 2π .

9. $y = \cot 3\theta$



10. $y = -\sec 2\theta$



11. $y = -2 \csc \frac{1}{2}\theta$

