Name ______________________ Class ____________ Date ____________

14-1 Reteaching

Trigonometric Identities

A *trigonometric identity* is a trigonometric equation that is true for all values of the variable except those that cause the expressions on either side of the equal sign to be undefined. You can use the trigonometric identities below to replace complicated-looking expressions with much simpler ones.

Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Problem

What is $\cot\theta(\tan\theta + \cot\theta)$ expressed in simplified terms?

Use the identities to rewrite $\cot\theta$ and $\tan\theta$.

$$\cot\theta(\tan\theta + \cot\theta) = \frac{1}{\tan\theta}\left(\tan\theta + \frac{1}{\tan\theta}\right)$$ Write $\cot\theta$ in terms of $\tan\theta$.

$$= \frac{1}{\tan\theta}(\tan\theta) + \left(\frac{1}{\tan\theta}\right)^2$$ Distribute.

$$= 1 + \left(\frac{1}{\tan\theta}\right)^2$$ Simplify.

$$= 1 + \cot^2\theta$$ Reciprocal Identity

$$= \csc^2\theta$$ Pythagorean Identity

Exercises

Simplify each expression.

1. $\cot\theta\sin\theta$ **cos θ**

2. $\tan\theta\cos\theta$ **sin θ**

3. $\csc\theta\sin\theta$ **1**

4. $\cos\theta\sin\theta\sec\theta$ **sin θ**

5. $\sin\theta + \cot\theta\cos\theta$ **csc θ**

6. $\csc^2\theta - \cot^2\theta$ **1**

14-1 Reteaching (continued)

Trigonometric Identities

To verify an identity, you can transform one side of the equation until it is the same as the other side. Begin by writing all of the functions in terms of sine and cosine.

Once you choose a side of the equation to transform, do not work with the other side of the equation. Raising both sides of the equation to a power or dividing both sides of the equation by a trigonometric expression can introduce extraneous solutions.

Problem

Verify the identity $1 + \cot^2\theta = \csc^2\theta$.

$$1 + \cot^2\theta = 1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 \quad \text{Cotangent Identity}$$

$$= 1 + \frac{\cos^2\theta}{\sin^2\theta} \quad \text{Simplify.}$$

$$= \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \quad \text{Write the fractions with common denominators.}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} \quad \text{Add.}$$

$$= \frac{1}{\sin^2\theta} \quad \text{Pythagorean Identity}$$

$$= \csc^2\theta \quad \text{Reciprocal Identity}$$

Exercises

Verify each identity. **7.–18. Answers may vary.**

7. $\cot\theta\tan\theta = 1$

8. $\cos\theta\sec\theta = 1$

9. $\csc\theta\sin\theta + \cot^2\theta = \csc^2\theta$

10. $\sin\theta(1 + \cot^2\theta) = \csc\theta$

11. $\sec\theta\cot\theta = \csc\theta$

12. $\sec^2\theta - \sec^2\theta\cos^2\theta = \tan^2\theta$

13. $\cot\theta\tan\theta + \tan^2\theta = \sec^2\theta$

14. $\csc^2\theta - \cot^2\theta = 1$

15. $\sin\theta + \cos\theta\cot\theta = \csc\theta$

16. $\frac{\sec\theta - \cos\theta}{\sec\theta} = \sin^2\theta$

17. $\cot\theta\sec\theta\sin\theta = 1$

18. $\tan\theta(\sin\theta - \csc\theta) = -\cos\theta$