Name ______________________________ Class ____________ Date ____________

13-7 Reteaching

Translating Sine and Cosine Functions

You can translate the graphs of sine and cosine functions both horizontally and vertically. A horizontal translation is called a *phase shift*. For a function in the form $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$:

- $|a| =$ amplitude
- $\frac{2\pi}{b} =$ period
- $h =$ phase shift — If $h > 0$, the graph moves to the right. If $h < 0$, the graph moves to the left.
- $k =$ vertical shift — If $k > 0$, the graph moves up. If $k < 0$, the graph moves down.

Problem

What are the amplitude, period, and any phase shift or vertical shift in the graph of the function $y = 2 \sin \frac{1}{3}(x + 5)$?

$y = 2 \sin \frac{1}{3}(x - (-5)) + 0$	Write function as $y = a \sin b(x - h) + k$.
$a = 2, b = \frac{1}{3}, h = -5, k = 0$	Identify a, b, h, and k.
$\lvert a \rvert = \lvert 2 \rvert = 2$	amplitude $= 2$
$\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{3}} = 6\pi$	period $= 6\pi$
$h = -5$	The phase shift is 5 units to the left.
$k = 0$	There is no vertical shift.

Exercises

Determine the amplitude, period, and any phase shift or vertical shift in the graphs of the functions.

1. $y = 6 \cos 3x + 2$

6; $\frac{2\pi}{3}$; 2 units up

2. $y = -\sin \frac{1}{2}(x - \pi)$

1; 4π; π units right

3. $y = 2 \sin 8\left(x - \frac{\pi}{3}\right) - 5$

2; $\frac{\pi}{4}$; $\frac{\pi}{3}$ right; 5 units down

4. $y = \cos 2(x - 1) + 3.4$

1; π; 1 unit right; 3.4 units up

5. $y = \frac{2}{3} \sin (x + 3\pi) - \pi$

$\frac{2}{3}$; 2π; 3π units left; π units down

6. $y = -3 \cos\left(x + \frac{\pi}{4}\right) + 12$

3; 2π; $\frac{\pi}{4}$ units left; 12 units up

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13-7 Reteaching (continued)

Translating Sine and Cosine Functions

The graph of a function in the form $y = a \sin b(x - h) + k$ is a translation of the graph of $y = a \sin bx$. The graph of a function in the form $y = a \cos b(x - h) + k$ is a translation of the graph of $y = a \cos bx$.

Problem

What is the graph of $y = 2 \sin 3\left(x - \frac{\pi}{3}\right) + 1$ in the interval from 0 to 2π?

Step 1 Compare the function to $y = a \sin b(x - h) + k$. $a = 2$ and $b = 3$
Find the amplitude, period, h, and k. $|a| = |2| = 2$; $\frac{2\pi}{3}$; $h = \frac{\pi}{3}$; $k = 1$

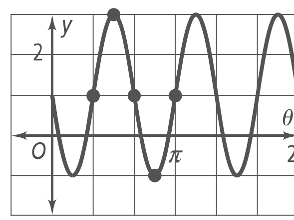
Step 2 Find the minimum and maximum of the curve before the vertical shift. Because the amplitude is 2, the maximum is 2 and the minimum is -2.

Step 3 Make a table of values. Choose x-values at intervals of one-fourth the period: $\frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$. The y-values before the vertical shift cycle through the pattern *zero-max-zero-min-zero*. Add h to the x-values and add k to the y-values to find the translated points.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$x + \frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
y	0	2	0	-2	0
$y + 1$	1	3	1	-1	1

Step 4 Plot the translated points from the table.

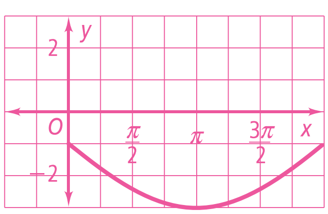
Step 5 Draw a smooth curve through the points. Extend the pattern from 0 to 2π.



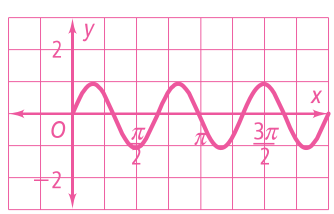
Exercises

Sketch each graph in the interval from 0 to 2π.

7. $y = -2 \sin \frac{1}{2}x - 1$



8. $y = \cos 3\left(x + \frac{\pi}{2}\right)$



9. $y = -2 \cos(x + \pi) - 2$