Name ________________________ Class ____________ Date ____________

6-3 Reteaching

Binomial Radical Expressions

Two radical expressions are *like radicals* if they have the same index and the same radicand.

Compare radical expressions to the terms in a polynomial expression.

Like terms:	$4x^3$	$11x^3$		The power and the variable are the same
Unlike terms:	$4y^3$	$11x^3$	$4y^2$	Either the power or the variable are not the same.
Like radicals:	$4\sqrt[3]{6}$	$11\sqrt[3]{6}$		The index and the radicand are the same
Unlike radicals:	$4\sqrt[3]{5}$	$11\sqrt[3]{6}$	$4\sqrt[2]{6}$	Either the index or the radicand are not the same.

When adding or subtracting radical expressions, simplify each radical so that you can find like radicals.

Problem

What is the sum? $\sqrt{63} + \sqrt{28}$

$\sqrt{63} + \sqrt{28} = \sqrt{9 \cdot 7} + \sqrt{4 \cdot 7}$ Factor each radicand.

$= \sqrt{3^2 \cdot 7} + \sqrt{2^2 \cdot 7}$ Find perfect squares.

$= \sqrt{3^2}\sqrt{7} + \sqrt{2^2}\sqrt{7}$ Use $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

$= 3\sqrt{7} + 2\sqrt{7}$ Use $\sqrt[n]{a^n} = a$ to simplify.

$= 5\sqrt{7}$ Add like radicals.

The sum is $5\sqrt{7}$.

Exercises

Simplify.

1. $\sqrt{150} - \sqrt{24}$ **$3\sqrt{6}$**

2. $\sqrt[3]{135} + \sqrt[3]{40}$ **$5\sqrt[3]{5}$**

3. $6\sqrt{3} - \sqrt{75}$ **$\sqrt{3}$**

4. $5\sqrt[3]{2} - \sqrt[3]{54}$ **$2\sqrt[3]{2}$**

5. $-\sqrt{48} + \sqrt{147} - \sqrt{27}$ **0**

6. $8\sqrt[3]{3x} - \sqrt[3]{24x} + \sqrt[3]{192x}$ **$10\sqrt[3]{3x}$**

6-3 Reteaching (continued)

Binomial Radical Expressions

- Conjugates, such as $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$, differ only in the sign of the second term. If a and b are rational numbers, then the product of conjugates produce a rational number:

  $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b.$
- You can use the conjugate of a radical denominator to rationalize the denominator.

Problem

What is the product? $\left(2\sqrt{7} - \sqrt{5}\right)\left(2\sqrt{7} + \sqrt{5}\right)$

$\left(2\sqrt{7} - \sqrt{5}\right)\left(2\sqrt{7} + \sqrt{5}\right)$ These are conjugates.

$= \left(2\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2$ Use the difference of squares formula.

$= 28 - 5 = 23$ Simplify.

Problem

How can you write the expression with a rationalized denominator? $\frac{4\sqrt{2}}{1 + \sqrt{3}}$

$\frac{4\sqrt{2}}{1 + \sqrt{3}}$

$= \frac{4\sqrt{2}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$ Use the conjugate of $1 + \sqrt{3}$ to rationalize the denominator.

$= \frac{4\sqrt{2} - 4\sqrt{6}}{1 - 3}$ Multiply.

$= \frac{4\sqrt{2} - 4\sqrt{6}}{-2} = -\frac{(4\sqrt{2} - 4\sqrt{6})}{2}$ Simplify.

$= \frac{-4\sqrt{2} + 4\sqrt{6}}{2} = -2\sqrt{2} + 2\sqrt{6}$

Exercises

Simplify. Rationalize all denominators.

7. $(3 + \sqrt{6})(3 - \sqrt{6})$ **3**

8. $\frac{2\sqrt{3} + 1}{5 - \sqrt{3}}$ $\frac{\sqrt{3} + 1}{2}$

9. $\left(4\sqrt{6} - 1\right)\left(\sqrt{6} + 4\right)$ $20 + 15\sqrt{6}$

10. $\frac{2 - \sqrt{7}}{2 + \sqrt{7}}$ $\frac{-11 + 4\sqrt{7}}{3}$

11. $\left(2\sqrt{8} - 6\right)\left(\sqrt{8} - 4\right)$ $40 - 28\sqrt{2}$

12. $\frac{\sqrt{5}}{2 + \sqrt{3}}$ $2\sqrt{5} - \sqrt{15}$