

# 6-4 Reteaching

## Rational Exponents

You can simplify a number with a rational exponent by converting the expression to a radical expression:

$$x^{\frac{1}{n}} = \sqrt[n]{x}, \text{ for } n > 0 \qquad 9^{\frac{1}{2}} = \sqrt[2]{9} = 3 \qquad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

You can simplify the product of numbers with rational exponents  $m$  and  $n$  by raising the number to the sum of the exponents using the rule

$$a^m \cdot a^n = a^{m+n}$$

### Problem

What is the simplified form of each expression?

a.  $36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}}$

$$\begin{aligned} 36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}} &= 36^{\frac{1}{4} + \frac{1}{4}} && \text{Use } a^m \cdot a^n = a^{m+n}. \\ &= 36^{\frac{1}{2}} && \text{Add.} \\ &= \sqrt[2]{36} && \text{Use } x^{\frac{1}{n}} = \sqrt[n]{x}. \\ &= 6 && \text{Simplify.} \end{aligned}$$

b. Write  $(6x^{\frac{2}{3}})(2x^{\frac{3}{4}})$  in simplified form.

$$\begin{aligned} (6x^{\frac{2}{3}})(2x^{\frac{3}{4}}) &= 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}} && \text{Commutative and Associative} \\ &&& \text{Properties of Multiplication} \\ &= 6 \cdot 2 \cdot x^{\frac{2}{3} + \frac{3}{4}} && \text{Use } x^m \cdot x^n = x^{m+n}. \\ &= 12x^{\frac{17}{12}} && \text{Simplify.} \end{aligned}$$

### Exercises

Simplify each expression. Assume that all variables are positive.

1.  $5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}$  **5**

2.  $(2y^{\frac{1}{4}})(3y^{\frac{1}{3}})$   **$6y^{\frac{7}{12}}$**

3.  $(-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}}$  **-11**

4.  $-y^{\frac{2}{3}} y^{\frac{1}{5}}$   **$-y^{\frac{13}{15}}$**

5.  $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}} \sqrt{5}$

6.  $(-3x^{\frac{1}{6}})(7x^{\frac{2}{6}})$   **$-21\sqrt{x}$**

# 6-4 Reteaching (continued)

## Rational Exponents

To write an expression with rational exponents in simplest form, simplify all exponents and write every exponent as a positive number using the following rules for  $a \neq 0$  and rational numbers  $m$  and  $n$ :

$$a^{-n} = \frac{1}{a^n} \quad \frac{1}{a^{-m}} = a^m \quad (a^m)^n = a^{mn} \quad (ab)^m = a^m b^m$$

### Problem

What is  $(8x^9y^{-3})^{-\frac{2}{3}}$  in simplest form?

$$(8x^9y^{-3})^{-\frac{2}{3}} = (2^3 x^9 y^{-3})^{-\frac{2}{3}}$$

Factor any numerical coefficients.

$$= (2^3)^{-\frac{2}{3}} (x^9)^{-\frac{2}{3}} (y^{-3})^{-\frac{2}{3}}$$

Use the property  $(ab)^m = a^m b^m$ .

$$= 2^{-2} x^{-6} y^2$$

Multiply exponents, using the property  $(a^m)^n = a^{mn}$ .

$$= \frac{y^2}{2^2 x^6}$$

Write every exponent as a positive number.

$$= \frac{y^2}{4x^6}$$

Simplify.

### Exercises

Write each expression in simplest form. Assume that all variables are positive.

7.  $(16x^2y^8)^{-\frac{1}{2}}$   $\frac{1}{4xy^4}$

8.  $(z^{-3})^{\frac{1}{9}}$   $\frac{1}{z^3}$

9.  $(2x^{\frac{1}{4}})^4$   $16x$

10.  $(25x^{-6}y^2)^{\frac{1}{2}}$   $\frac{5y}{x^3}$

11.  $(8a^{-3}b^9)^{\frac{2}{3}}$   $\frac{4b^6}{a^2}$

12.  $\left(\frac{16z^4}{25x^8}\right)^{-\frac{1}{2}}$   $\frac{5x^4}{4z^2}$

13.  $\left(\frac{x^2}{y^{-1}}\right)^{\frac{1}{5}}$   $x^{\frac{2}{5}}y^{\frac{1}{5}}$

14.  $(27m^9n^{-3})^{-\frac{2}{3}}$   $\frac{n^2}{9m^6}$

15.  $\left(\frac{32r^2}{2s^4}\right)^{\frac{1}{4}}$   $\frac{2r^{\frac{1}{2}}}{s}$

16.  $(9z^{10})^{\frac{3}{2}}$   $27z^{15}$

17.  $(-243)^{-\frac{1}{5}}$   $-\frac{1}{3}$

18.  $\left(\frac{x^{\frac{2}{5}}}{y^{\frac{1}{2}}}\right)^{10}$   $\frac{x^4}{y^5}$