Name ______________________ Class ____________ Date ____________

6-6 Reteaching

Function Operations

When you combine functions using addition, subtraction, multiplication, or division, the domain of the resulting function has to include the domains of both of the original functions.

Problem

Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$. What is the solution of each function operation? What is the domain of the result?

a. $(f + g)(x) = f(x) + g(x) = (x^2 - 4) + (\sqrt{x}) = x^2 + \sqrt{x} - 4$

b. $(f - g)(x) = f(x) - g(x) = (x^2 - 4) - (\sqrt{x}) = x^2 - \sqrt{x} - 4$

c. $(g - f)(x) = g(x) - f(x) = (\sqrt{x}) - (x^2 - 4) = -x^2 + \sqrt{x} + 4$

d. $(f \cdot g)(x) = f(x) \cdot g(x) = (x^2 - 4)(\sqrt{x}) = x^2\sqrt{x} - 4\sqrt{x}$

The domain of f is all real numbers. The domain of g is all $x \geq 0$. For parts a–d, there are no additional restrictions on the values for x, so the domain for each of these is $x \geq 0$.

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{\sqrt{x}} = \frac{(x^2 - 4)\sqrt{x}}{x}$

As before, the domain is $x \geq 0$. But, because the denominator cannot be zero, eliminate any values of x for which $g(x) = 0$. The only value for which $\sqrt{x} = 0$ is $x = 0$. Therefore, the domain of $\frac{f}{g}$ is $x > 0$.

f. $\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x}}{x^2 - 4}$

Similarly, begin with $x \geq 0$ and eliminate any values of x that make the denominator $f(x)$ zero: $x^2 - 4 = 0$ when $x = -2$ and $x = 2$. Therefore, the domain of $\frac{g}{f}$ is $x \geq 0$ combined with $x \neq -2$ and $x \neq 2$. In other words, the domain is $x \geq 0$ and $x \neq 2$, or all nonnegative numbers except 2.

Exercises

Let $f(x) = 4x - 3$ and $g(x) = x^2 + 2$. Perform each function operation and then find the domain of the result.

1. $(f + g)(x)$

$x^2 + 4x - 1$; all real numbers

2. $(f - g)(x)$

$-x^2 + 4x - 5$; all real numbers

3. $(g - f)(x)$

$x^2 - 4x + 5$; all real numbers

4. $(f \cdot g)(x)$

$4x^3 - 3x^2 + 8x - 6$; all real numbers

5. $\frac{f}{g}(x)$

$\frac{4x - 3}{x^2 + 2}$; all real numbers

6. $\frac{g}{f}(x)$

$\frac{x^2 + 2}{4x - 3}$; $x \neq \frac{3}{4}$

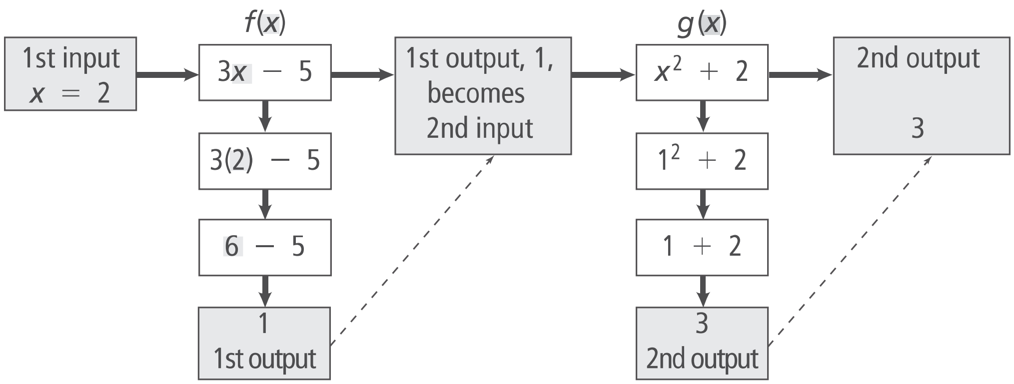
6-6 Reteaching (continued)

Function Operations

- One way to combine two functions is by forming a composite.
- A composite is written $(g \circ f)$ or $g(f(x))$. The two different functions are g and f.
- Evaluate the inner function $f(x)$ first.
- Use this value, the first output, as the input for the second function, $g(x)$.

Problem

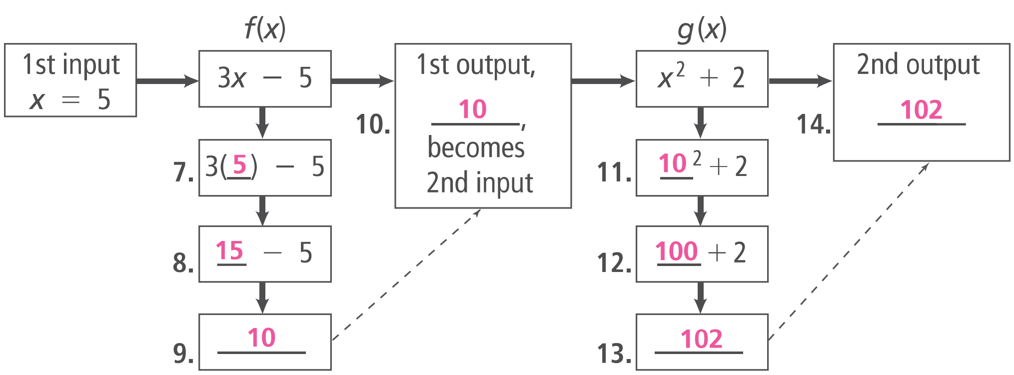
What is the value of the expression $g(f(2))$ given the inner function, $f(x) = 3x - 5$ and the outer function, $g(x) = x^2 + 2$?



Exercises

Evaluate the expression $g(f(5))$ using the same functions for g and f as in the Example. Fill in blanks 7–14 on the chart.

Use one color highlighter to highlight the first input. Use a second color to highlight the first output and the second input. Use a third color to highlight the second output, which is the answer.



Given $f(x) = x^2 + 4x$ and $g(x) = 2x + 3$, evaluate each expression.

15. $f(g(2))$ 77

16. $g(f(2.5))$ 35.5

17. $g(f(-5))$ 13

18. $f(g(-5))$ 21