

## Chapter 8 Rational Functions

### Inverse Variation

$xy = k$ ,  $y = \frac{k}{x}$ , or  $x = \frac{k}{y}$ , where  $k \neq 0$ .

### Combined Variation

$z$  varies jointly with  $x$  and  $y$ :  $z = kxy$

$z$  varies jointly with  $x$  and  $y$  and inversely with  $w$ :  $z = \frac{kxy}{w}$

$z$  varies directly with  $x$  and inversely with the product  $wy$ :  $z = \frac{kx}{wy}$

### Reciprocal Functions

Parent  $y = \frac{1}{x}$ ,  $x \neq 0$

Reflection across  $x$ -axis  $y = -\frac{1}{x}$ ,  $x \neq 0$

Stretch ( $a > 1$ )  $y = \frac{a}{x}$ ,  $x \neq 0$

Shrink ( $0 < a < 1$ )

Translation

horizontal by  $h$

vertical by  $k$

$$y = \frac{a}{x-h} + k, x \neq h$$

Asymptotes

$$y = k \text{ (horiz.)}, x = h \text{ (vert.)}$$

The **rational function**  $f(x) = \frac{P(x)}{Q(x)}$  has a **point of discontinuity** for each real zero of  $Q(x)$ .

### Operations With Rational Expressions

A *rational expression* is an expression that can be written in the form  $\frac{\text{polynomial}}{\text{polynomial}}$ , where the denominator is not zero. A rational expression is in simplest form if the numerator and denominator have no common factors except 1.

To add or subtract two rational expressions, use a common denominator.

To multiply rational expressions, first find and divide out any common factors in the numerators and the denominators. Then multiply the remaining numerators and denominators. To divide rational expressions, first use a reciprocal to change the problem to multiplication.