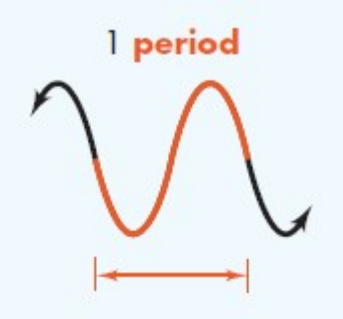
ALGEBRA 2 B – UNIT 7 – PERIODIC FUNCTIONS & TRIGONOMETRY

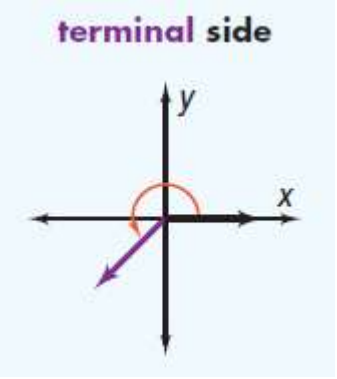
Math Usage: The period of a function is the horizontal length of one cycle. A *periodic function* is a function that repeats a pattern of y-values at regular intervals. The *amplitude* of a periodic function measures the amount of variation in the function values.



The **midline** is the horizontal line midway between the maximum and minimum values of a periodic function. The **amplitude** is half the difference between the maximum and minimum values of the function.

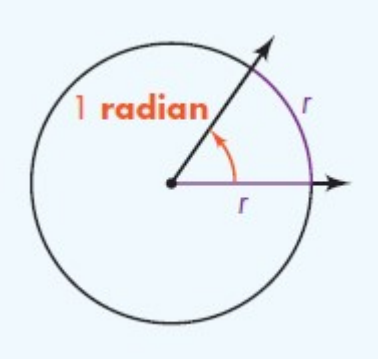
$$\text{amplitude} = \frac{1}{2}(\text{maximum value} - \text{minimum value})$$

Math Usage: An angle in standard position is formed by rotating a ray from the x-axis about its endpoint. The final position of the ray is the **terminal side** of the angle. Two angles in standard position are *coterminal* angles if they have the same **terminal side**.



All content from Algebra 2, 2011, Prentice Hall, Pearson.

Math Usage: Radians measure the amount of rotation from the initial side to the terminal side of an angle. An angle with a full-circle rotation measures 2π radians.



Convert Between Radians and Degrees

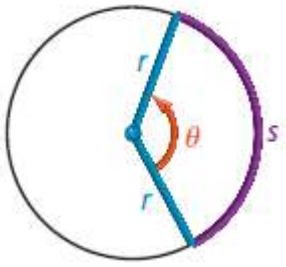
Use the proportion $\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$ to convert between radians and degrees.

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure θ (in radians), the length s of the intercepted arc is $s = r\theta$.

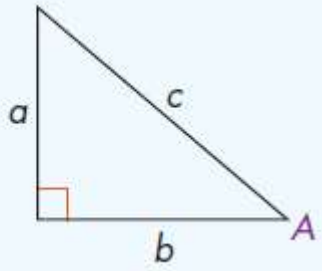


Main Idea: Trigonometry relates the measures of angles of a triangle to the lengths of its sides.

$\sin A = \frac{a}{c}$

$\cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$

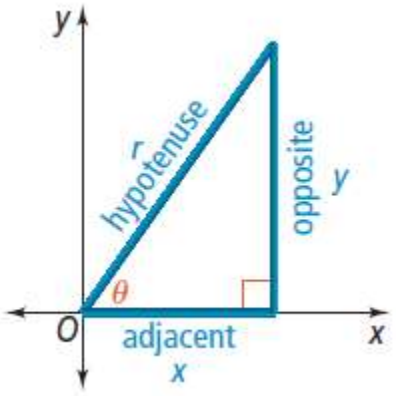


Remember: SOH-CAH-TOA

$$\sin\theta = \frac{y}{r} = \frac{\text{OPP}}{\text{HYP}} \qquad \csc\theta = \frac{r}{y} = \frac{\text{HYP}}{\text{OPP}}$$

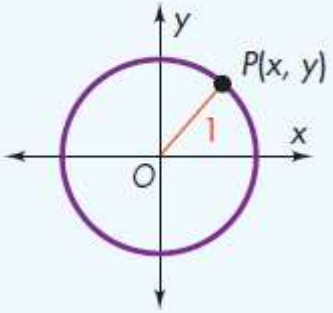
$$\cos\theta = \frac{x}{r} = \frac{\text{ADJ}}{\text{HYP}} \qquad \sec\theta = \frac{r}{x} = \frac{\text{HYP}}{\text{ADJ}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{OPP}}{\text{ADJ}} \qquad \cot\theta = \frac{x}{y} = \frac{\text{ADJ}}{\text{OPP}}$$



Definition: A unit circle is a circle with radius 1 centered at (0, 0) on the coordinate plane.

unit circle



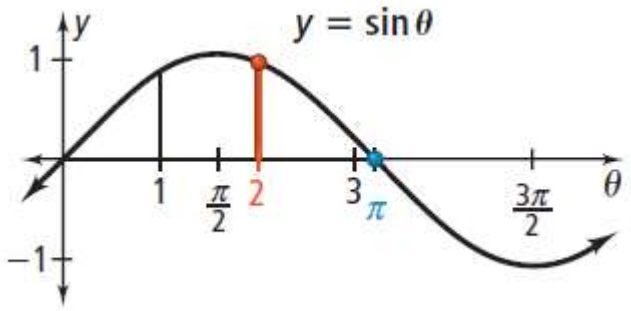
Main Idea: For any point (x, y) on the unit circle, a right triangle can be formed with a hypotenuse of length 1 and side lengths x and y. Using the definition of sine and cosine, $x = \cos\theta$, and $y = \sin\theta$.

Definition: If the terminal side of an angle θ in standard position intersects the unit circle at the point (x, y), then the sine of θ is the y-coordinate of the point (x, y).

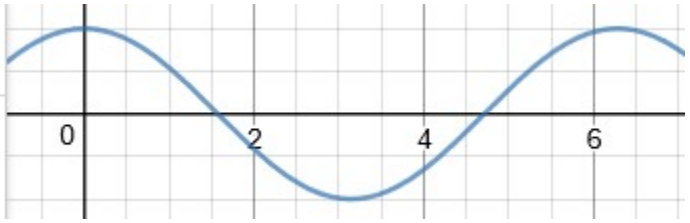
Definition: If the terminal side of an angle θ in standard position intersects the unit circle at the point (x, y), then the cosine of θ is the x-coordinate of the point (x, y).

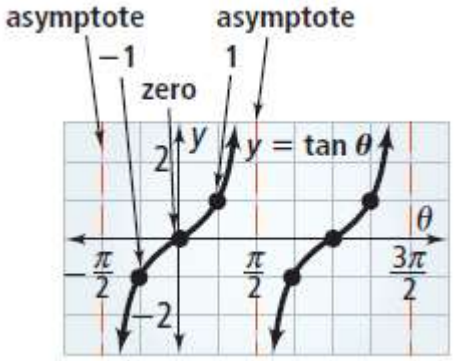
Definition: If the terminal side of an angle θ in standard position intersects the unit circle at the point (x, y) then the tangent of θ is the ratio of the y-coordinate to the x-coordinate, $\frac{y}{x}$.

Function	Domain	Range	Inverse Function	Domain	Range
$y = \cos\theta$	$0 \le \theta \le \pi$	$-1 \le y \le 1$	$\theta = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$y = \sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$-1 \le y \le 1$	$\theta = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$y = \tan\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	y is any real number	$\theta = \tan^{-1} x$	x is any real number	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



$y = \cos(x)$





Sine and Cosine Functions

	Sine	Cosine
Parents	$y = \sin x$	$y = \cos x$
Reflection across x-axis	$y = -\sin x$	$y = -\cos x$
Amplitude $\lvert a \rvert$	$y = a \sin x$	$y = a \cos x$
Period $\frac{2\pi}{b}$, $b > 0$	$y = \sin bx$	$y = \cos bx$
Translation horizontal by h vertical by k	$y = \sin (x - h) + k$	$y = \cos (x - h) + k$

Tangent Function

Parent	$y = \tan x$
Reflection across x-axis	$y = -\tan x$
Period $\frac{\pi}{b}$	$y = \tan bx$
Translation horizontal by h vertical by k	$y = \tan (x - h) + k$
Asymptotes ($\tan bx$)	$x = n\frac{\pi}{2b}$, n odd

Basic Identities

Reciprocal Identities:

$\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\tan \theta = \frac{1}{\cot \theta}$

$\sin \theta = \frac{1}{\csc \theta}$ $\cos \theta = \frac{1}{\sin \theta}$ $\cot \theta = \frac{1}{\tan \theta}$

Tangent Identity: $\tan \theta = \frac{\sin \theta}{\cos \theta}$

Cotangent Identity: $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean Identities

$\cos^2 \theta + \sin^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$