UNIT 4 GRAPHS

PRECALCULUS B

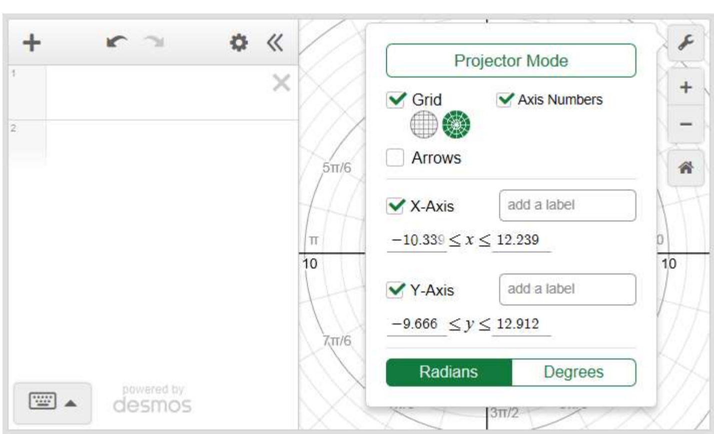
LESSONS:

- 3: Graphs of Polar Equations
- 4: Conic Sections in Polar Coordinates
- 5: Limaçons – 4 types
- 6: Rose Curves & Lemniscates

A Note about Graphing in Polar Coordinates . . .

DESMOS!!!

YES! !
Desmos can graph in Polar Coordinates!

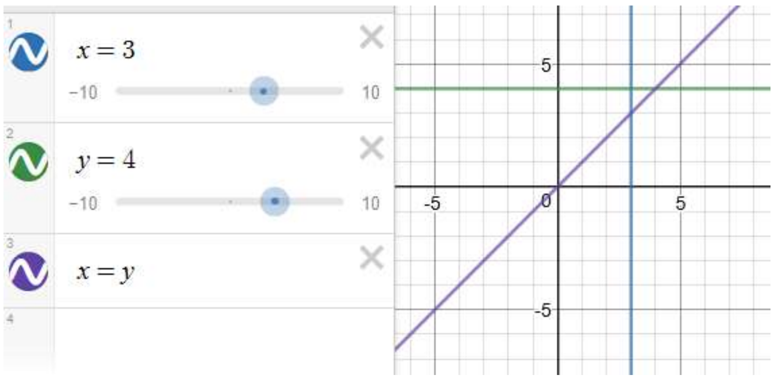


Go to the "wrench" to change the "Grid".

Also, remember to check whether you are working in radians or degrees ☺

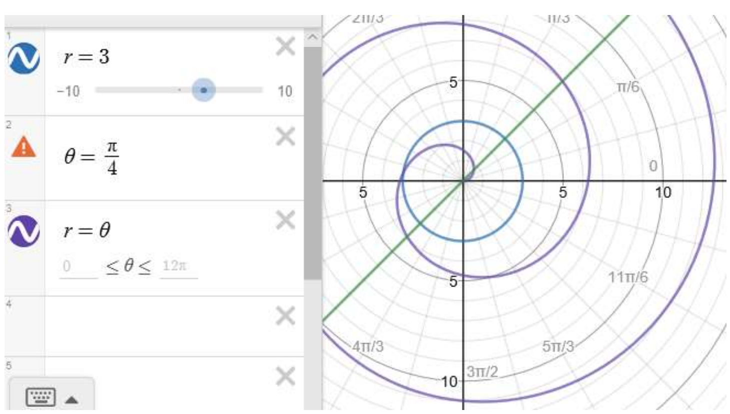
A comparison of graphing in Rectangular vs Polar Coordinates

Rectangular Coordinates (x, y)



A comparison of graphing in Rectangular vs Polar Coordinates

Polar Coordinates (r, θ)



Types of Polar Graphs we are using:

- Conic Sections
- Limaçons (4 types)
- Rose Curves
- Lemniscates

Types of Polar Graphs we are using:

How to pronounce Limaçons

(`lim-a-con) or (le-ma-`son)

How to pronounce Lemniscate

(lem-`ni-skit) or (`lem-ni-skate)

Types of Polar Graphs we are using:

Note:
Each of these equation types comes in a sine version and a cosine version.

And:
Each equation type comes in a positive and negative version.

IMPORTANT:

Remember, that on the Unit Circle the points around the edge are (x, y) coordinates.

And, for the Unit Circle,
the x-coordinate was the cosine of θ,
and the y-coordinate was the sine of θ.

So … cosine will correspond to the horizontal, and sine will correspond to the vertical!!

CONIC SECTIONS IN POLAR EQUATIONS

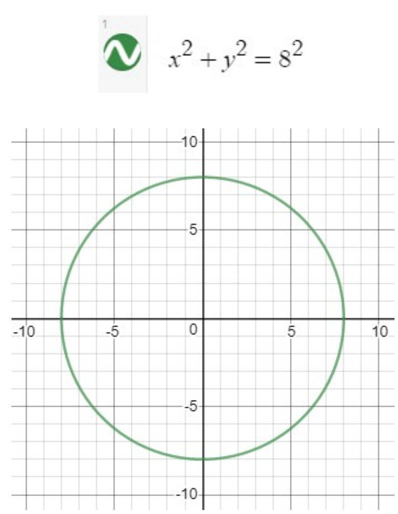
- **Circles**
- **Ellipses**
- **Parabolas**
- **Hyperbolas**

A circle in Rectangular vs Polar

Rectangular Coordinates (x, y)

$x^2 + y^2 = r^2$ . . . centered at the origin

$(x - h)^2 + (y - k)^2 = r^2$ . . . shifted

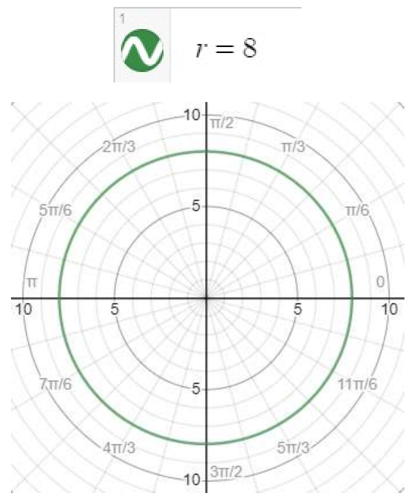


A circle in Rectangular vs Polar

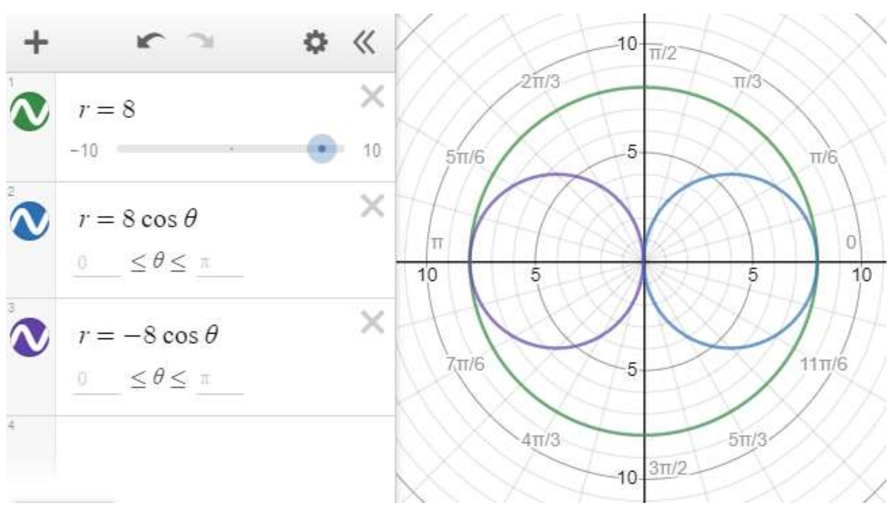
Polar Coordinates (r, θ)

r = a . . . centered at the pole

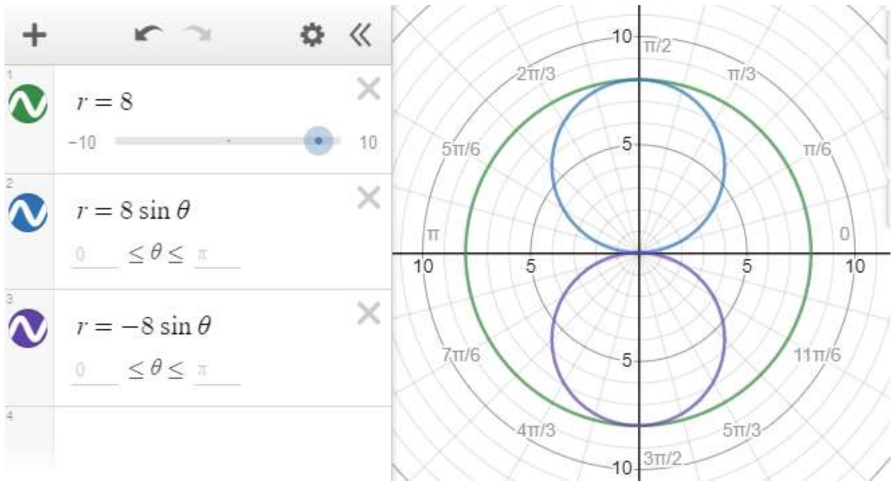
r = ±a cosθ or r = ±a sinθ . . . shifted



Shifting Polar Circles . . . The cosine version

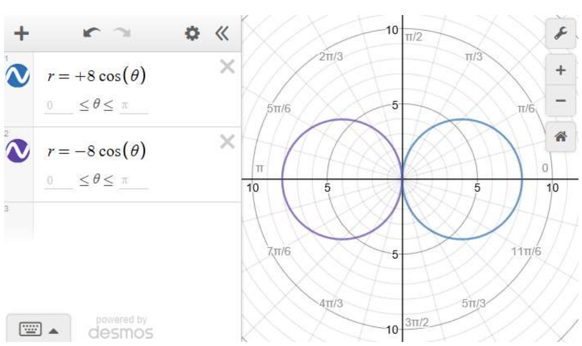


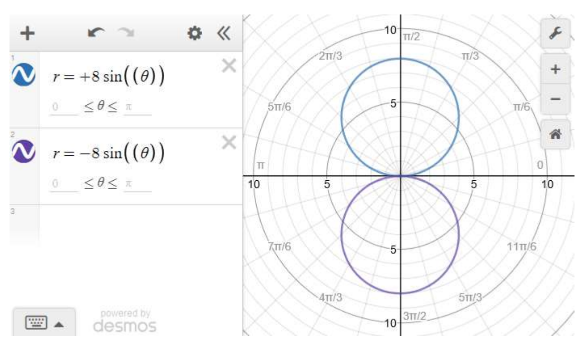
Shifting Polar Circles . . . The sine version



Shifting Polar Circles . . .

What patterns do you notice?





Shifting Polar Circles . . .

What patterns do you notice?

- Cosine shifts along the horizontal.
- Sine shifts along the vertical.
- The shift is only to where the edge of the circle is at the pole.
- The negative version is a reflection to either left (cos) or down (sin).
- The number goes from being the diameter to being the radius.

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

*Yes, they are all the same equation!!

Ellipse: If $0 < e < 1$
Parabola: If $e = 1$
Hyperbola: If $e > 1$

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

"e" is the "eccentricity", a measurement of how far it deviates from a circle
"d" is the distance from the focus point to the directrix line
And notice that the denominator always starts with the number 1.

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

TRY IT: What are e and d?

$$r = \frac{8}{2+4\cos\theta}$$

Make it match the pattern!

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

$$r=\frac{ed}{1\pm e\cos\theta} \text{ or } r=\frac{ed}{1\pm e\sin\theta}$$

TRY IT: What are e and d?

$$r=\frac{8}{2+4\cos\theta}$$

Divide each part by 2 to match the pattern that the denominator starts with 1.

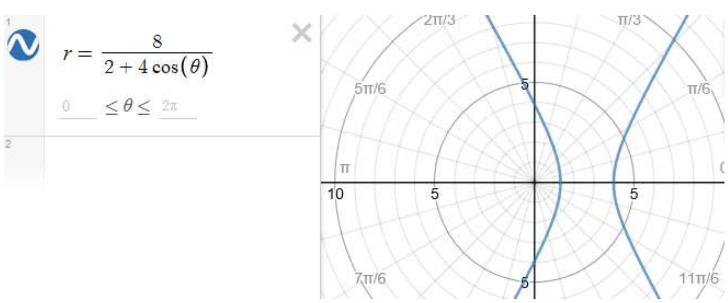
$$r=\frac{4}{1+2\cos\theta}$$

Look at the pattern, e is 2.

Next, ed=4, and e=2, so d=2.

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:



Look at the pattern, e is 2.

SO, e>1 makes this a hyperbola!

Ellipse: If 0 < e < 1
Parabola: If e = 1
Hyperbola: If e > 1

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

TRY IT: Name the Conic Sections & don't use Desmos!

$$r = \frac{4}{2 - \cos\theta}$$

$$r = \frac{6}{3 + 9\sin\theta}$$

$$r = \frac{3}{1 + \cos\theta}$$

$$r = \frac{ed}{1 \pm e\cos\theta} \text{ or } r = \frac{ed}{1 \pm e\sin\theta}$$

Ellipse: If $0 < e < 1$
Parabola: If $e = 1$
Hyperbola: If $e > 1$

The other Conic Sections . . .

Ellipse, Parabola, Hyperbola:

TRY IT: Name the Conic Sections

$$r = \frac{4}{2 - \cos\theta}$$

Divide by 2 … e = ½

Ellipse

$$r = \frac{6}{3 + 9\sin\theta}$$

Divide by 3 … e = 3

Hyperbola

$$r = \frac{3}{1 + \cos\theta}$$

Divide by 1 … e = 1

Parabola

$$r = \frac{ed}{1 \pm e\cos\theta} \text{ or } r = \frac{ed}{1 \pm e\sin\theta}$$

Ellipse: If $0 < e < 1$
Parabola: If $e = 1$
Hyperbola: If $e > 1$

LIMACONS IN POLAR EQUATIONS

- Inner Loop
- Cardiod
- Dimple
- Convex

Limaçons . . . The "dented" circles

$r = a \pm b \sin\theta$ and $r = a \pm b \cos\theta$

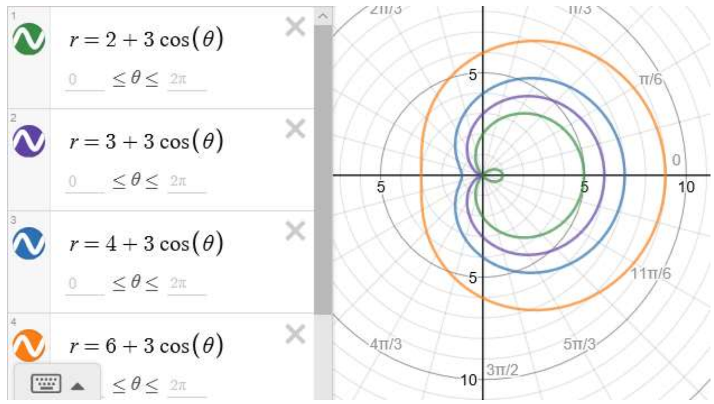
- Inner Loop
- Cardiod
- Dimple
- Convex

✓ $a/b < 1$
✓ $a/b = 1$
✓ $1 < a/b < 2$
✓ $a/b \geq 2$

Limaçons . . . The "dented" circles

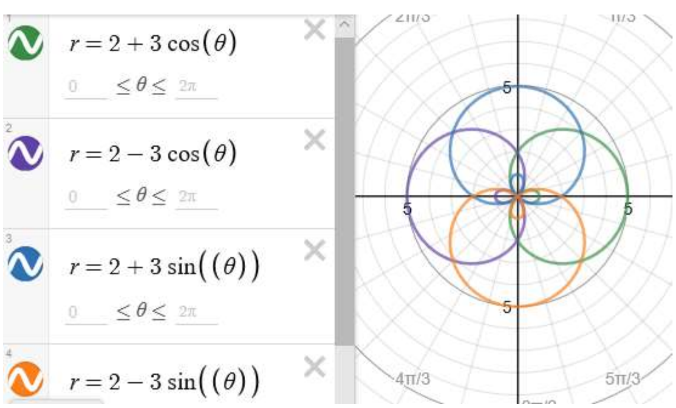
$r = a \pm b\sin\theta$ and $r = a \pm b\cos\theta$

Inner Loop	❑ a/b <1
Cardiod	❑ a/b = 1
Dimple	❑ 1< a/b <2
Convex	❑ a/b ≥ 2



Limaçons . . . The "dented" circles

$r = a \pm b\sin\theta$ and $r = a \pm b\cos\theta$



Sin vs Cos

Remember, cosθ is horizontal, sinθ is vertical, and minus reflects.

Limaçons . . . The “dented” circles

$$r = a \pm b\sin\theta \text{ and } r = a \pm b\cos\theta$$

TRY IT: Name the type & Name the orientation (up, down, left, right)

1. $r = 7 + 3\cos(\theta)$, $0 \le \theta \le 2\pi$
2. $r = 1 - 3\cos(\theta)$, $0 \le \theta \le 2\pi$
3. $r = 4 + 3\sin(\theta)$, $0 \le \theta \le 2\pi$
4. $r = 3 - 3\sin(\theta)$

Limaçons . . . The “dented” circles

$$r = a \pm b\sin\theta \text{ and } r = a \pm b\cos\theta$$

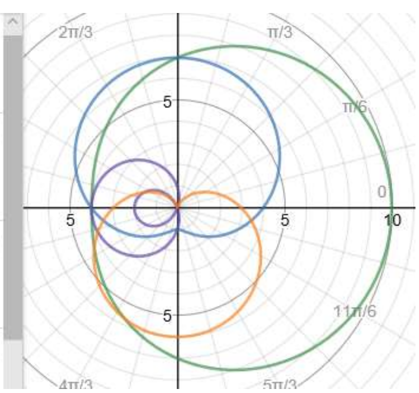
Convex & right

Inner Loop & left

Dimple & up

Cardiod & down

1. $r = 7 + 3\cos(\theta)$, $0 \le \theta \le 2\pi$
2. $r = 1 - 3\cos(\theta)$, $0 \le \theta \le 2\pi$
3. $r = 4 + 3\sin(\theta)$, $0 \le \theta \le 2\pi$
4. $r = 3 - 3\sin(\theta)$



Limaçons . . . The "dented" circles

$r = a \pm b\sin\theta$ and $r = a \pm b\cos\theta$

- Inner Loop ✓ $a/b < 1$
- Cardiod ✓ $a/b = 1$
- Dimple ✓ $1 < a/b < 2$
- Convex ✓ $a/b \geq 2$

NOTE: See the charts in the Key Concepts document for the patterns that the horizontal and vertical intercepts follow for each of these shapes.

ROSE CURVES IN POLAR EQUATIONS

Rose Curves . . . "flowers" ☺

*This time, instead of adding to the basic circle equation, we will multiply the angle!

$r = a \sin n\theta$ and $r = a \cos n\theta$

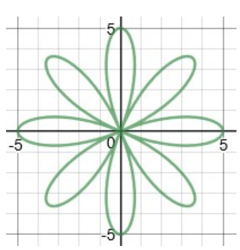
"a" is the length of each "petal"
"n" tells you how many petals

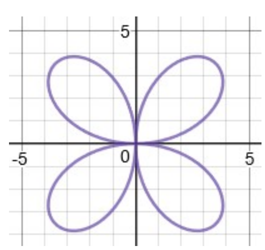
BUT, be careful!
If n is even, double n to get the number of petals
If n is odd, then n equals the number of petals

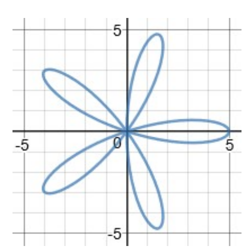
Rose Curves . . . "flowers" ☺

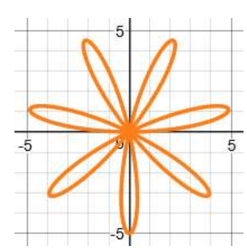
$r = a \sin n\theta$ and $r = a \cos n\theta$

TRY IT: For these, every petal has a length of 5.
But what is n? And is it sin or cos?







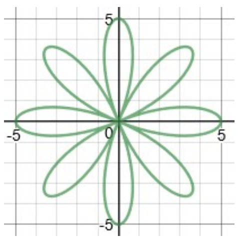


NOTE: A cosine Rose Curve will have a petal along the horizontal axis.
A sine Rose Curve will have vertical symmetry.

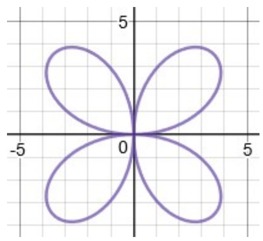
Rose Curves . . . “flowers” ☺

$r = a \sin n\theta$ and $r = a \cos n\theta$

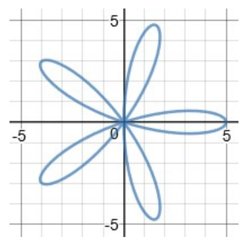
For these, every petal has a length of 5.
But what is n? And is it sin or cos?



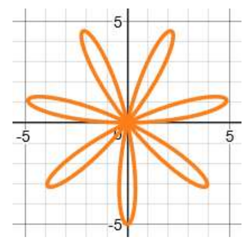
$r = 5\cos(4\theta)$



$r = 5\sin(2\theta)$



$r = 5\cos(5\theta)$



$r = 5\sin(7\theta)$

LEMNISCATES IN POLAR EQUATIONS

Lemniscates . . . “infinity” or “figure 8”

$$r^2 = a^2 \sin 2\theta \text{ and } r^2 = a^2 \cos 2\theta$$

***This time, instead of adding or a single multiplying, we do some squaring and doubling!**

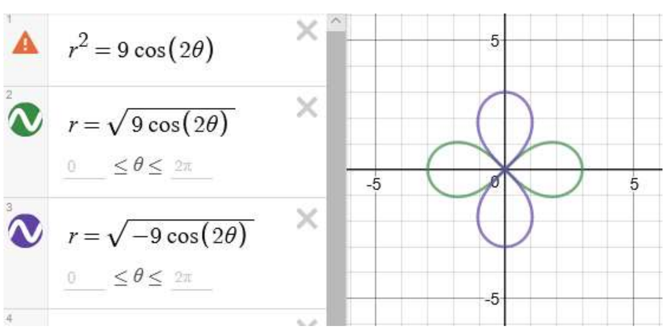
The positive cosine version is along the horizontal, BUT the vertical is not the sine version this time!

The negative cosine is the vertical!

The sine version goes along the diagonals.

Lemniscates . . . “infinity” or “figure 8”

$$r^2 = a^2 \sin 2\theta \text{ and } r^2 = a^2 \cos 2\theta$$



Note: Desmos doesn’t like starting with r^2
So, square root both sides!

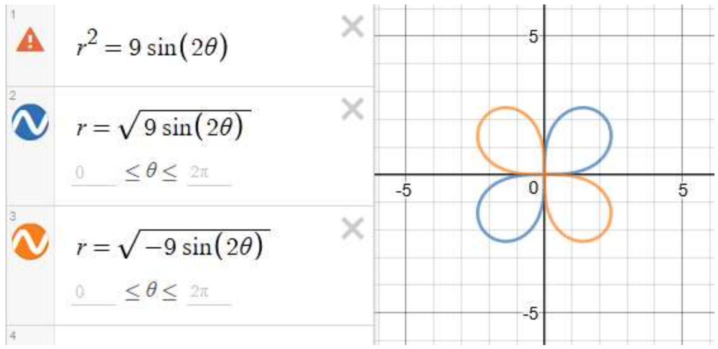
Also, notice that these are BOTH cosine, even though one is vertical!

Lemniscates . . . "infinity" or "figure 8"

$r^2 = a^2 \sin 2\theta$ and $r^2 = a^2 \cos 2\theta$

Notice that the positive sine is along the diagonal with a positive slope.

And the negative sine is along the diagonal with a negative slope.



POLAR EQUATION GRAPHS:

Conic Sections

Circle

$r = a \cos \theta$

$r = a \sin \theta$

Ellipse
Parabola
Hyperbola

$$r = \frac{ed}{1 \pm e\cos\theta}$$

$$r = \frac{ed}{1 \pm e\sin\theta}$$

Limaçons

Inner Loop
Cardiod
Dimple
Convex

$r = a \pm b\cos\theta$

$r = a \pm b\sin\theta$

Rose Curves

even petals
odd petals

$r = a \cos n\theta$

$r = a \sin n\theta$

Lemniscates

along the axes
along the diagonals

$r^2 = a^2 \cos 2\theta$

$r^2 = a^2 \sin 2\theta$

Use the tools!

- ❑ Key Words document
- ❑ Key Concepts document
- ❑ DESMOS

☺ And, just for fun, play around with changing parts of these equations to see what other fun shapes you might get!