UNIT 4 LESSONS 5-6

PRECALCULUS A

LESSONS:

- Graphs of Rational Functions
- Operations with Rational Functions

What is a Rational Function???

What is a Rational Function???

… Rational uses the word Ratio

… Ratios are written as Fractions

… so a Rational Function is a fraction full of algebra!!!

… specifically, a fraction of Polynomials!

But fractions can get messy!

… Yep ☺

Just carefully follow the processes you learned for fractions.

1^{st}

Denominators are
NOT allowed
to equal zero!!

1st

Denominators are NOT allowed to equal zero!!

& any value for x that would make a denominator = 0 becomes a restriction on the domain

What is the domain??

$$g(x) = \frac{x+1}{x^2 - 3x - 10}$$

What is the domain??

$$g(x) = \frac{x + 1}{x^2 - 3x - 10}$$

The domain is any value for x,
except what would make the denominator equal to zero.

What is the domain??

$$g(x) = \frac{x + 1}{x^2 - 3x - 10}$$

So, set the denominator equal to zero and solve for x.

$$x^2 - 3x - 10 = 0$$

What is the domain??

$$g(x) = \frac{x+1}{x^2 - 3x - 10}$$

$$x^2 - 3x - 10 = 0$$
$$(x+2)(x-5) = 0$$
$$x = -2 \text{ or } x = 5$$

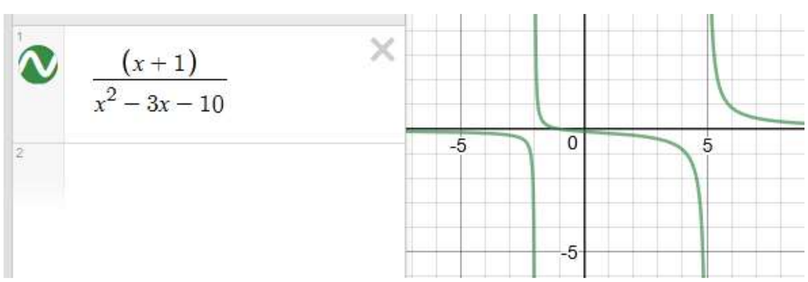
So, the Domain is:

$$(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$$

Now see it on the graph:

$$g(x) = \frac{x+1}{x^2 - 3x - 10}$$

$$(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$$



Wherever there is a break in the domain,

it is either a _____________ _____________,

or a _____________ _____________.

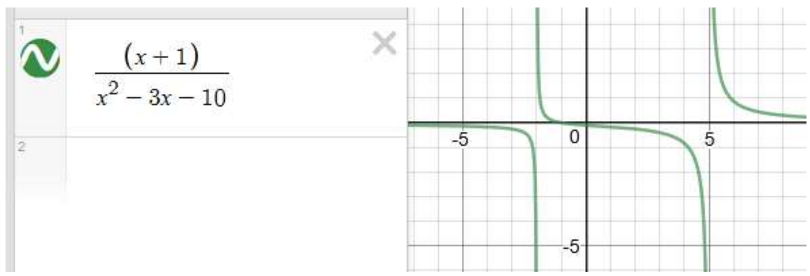
Wherever there is a break in the domain,

it is either a vertical asymptote,

or a removable discontinuity.

Vertical Asymptotes at x=-2 and at x=5

$$g(x) = \frac{x+1}{x^2 - 3x - 10} \qquad (-\infty,-2) \cup (-2,5) \cup (5,\infty)$$



But when you factor the function and it has common factors than can be cancelled out . . .

$$f(x) = \frac{8x-8}{x^2+3x-4} \quad \Rightarrow \quad f(x) = \frac{8(x-1)}{(x-1)(x+4)}$$

You get ______________________________

But when you factor it and it has common factors than can be cancelled out . . .

$$f(x) = \frac{8x-8}{x^2+3x-4} \Rightarrow f(x) = \frac{8(x-1)}{(x-1)(x+4)}$$

You get . . .

- Domain restrictions at the zeros of the denominator
- A removable discontinuity at the cancelled factor
- A vertical asymptote at the remaining denominator factor

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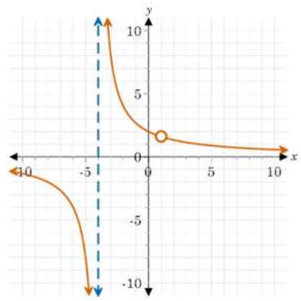
$$f(x) = \frac{8x-8}{x^2+3x-4} \Rightarrow f(x) = \frac{8(x-1)}{(x-1)(x+4)}$$

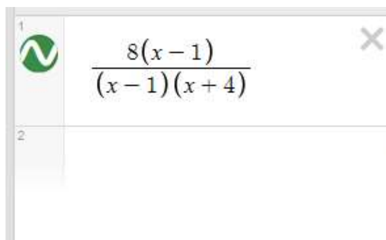
You get . . .

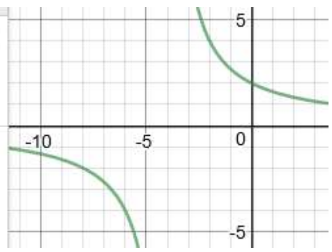
- Domain restrictions at the zeros of the denominator
- A vertical asymptote at the remaining denominator factor
- A removable discontinuity at the cancelled factor

NOTE: The removable discontinuity WON'T show on Desmos!
. . . But you need to know it is there!



$$f(x) = \frac{8(x-1)}{(x-1)(x+4)}$$





One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

What are the restrictions on the domain?

How do you write this domain?

Are there any removable discontinuities?

Are there any vertical asymptotes?

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

1st Factor!

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

1st Factor!

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

Next, set each denominator factor equal to zero, and solve for x.

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

Next, set each <u>denominator</u> factor equal to zero, and solve for x.

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

These are the restrictions on the domain!

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

Domain: $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, 0\right) \cup (0,3) \cup (3,\infty)$

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

Which restrictions are removable discontinuities, and which are vertical asymptotes??

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

Removable discontinuities happen at cancelled factors, so, there is a removable discontinuity at __________.
Vertical asymptote lines happen at the remaining denominator factors, so there are vertical asymptotes at __________.

One more time . . .

$$f(x) = \frac{3x^2 - 12x + 9}{2x^3 - x^2 - 15x}$$

$$= \frac{3(x-3)(x-1)}{x(2x+5)(x-3)}$$

$$x = 0,\ x = -\frac{5}{2},\ \text{or } x = 3.$$

Removable discontinuities happen at cancelled factors, so, there is a removable discontinuity at x=3.
Vertical asymptote lines happen at the remaining denominator factors, so there are vertical asymptotes at x=0 and x=-5/2.

Then there are horizontal asymptotes . . .

Key Concept

Finding Horizontal Asymptotes

The rational function $f(x) = \frac{p(x)}{q(x)}$, where the degree of $p(x)$ is n and the degree of $q(x)$ is m, has the following characteristics:

- If $n < m$, then the function has a horizontal asymptote at $y = 0$.
- If $n = m$, then the function has a horizontal asymptote at $y = \frac{a}{b}$, where a is the leading coefficient of p and b is the leading coefficient of q.
- If $n > m$, then there is no horizontal asymptote.

Then there are horizontal asymptotes . . .

In other words . . .

Then there are horizontal asymptotes . . .

1st Write the numerator and denominator in standard form (descending powers).

2nd Compare the degrees (highest power) of the numerator and denominator.

Then there are horizontal asymptotes . . .

IF the top degree is smaller than the bottom degree, then the horizontal asymptote is the line y=O.

IF the degrees are the same, then the horizontal asymptote is the line y= the fraction made by the leading coefficients.

IF the top degree is larger than the bottom degree, then there is no horizontal asymptote line.

PRACTICE:

$$f(x) = \frac{x^2}{x-6}$$

Degree?
Degree?

Horizontal asymptote?

PRACTICE:

$$f(x) = \frac{x^2}{x-6}$$

Degree 2
Degree 1

Horizontal asymptote:

Top is larger, so none.

PRACTICE:

$$g(x) = \frac{2x^3 + 7}{x^4 - 3x^2 + 2}$$

Degree?
Degree?

Horizontal asymptote?

PRACTICE:

$$g(x) = \frac{2x^3 + 7}{x^4 - 3x^2 + 2}$$

Degree 3
Degree 4

Horizontal asymptote:

Top is smaller, so asymptote at y=0.

PRACTICE:

$$h(x) = \frac{3x^2 - 1}{4x^2 - 2x - 5}$$

Degree?
Degree?

Horizontal asymptote?

PRACTICE:

$$h(x) = \frac{3x^2 - 1}{4x^2 - 2x - 5}$$

Degree 2
Degree 2

Horizontal asymptote:

Same, so asymptote at y=3/4

NOTE:

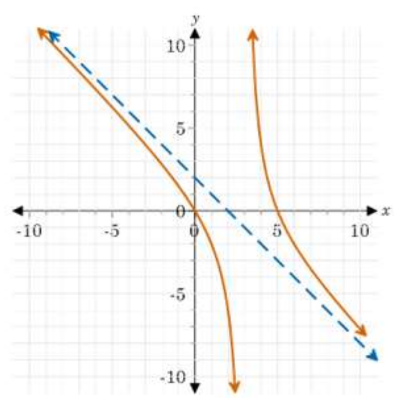
For this pattern, double check
if it is in standard form.

And always watch the signs.

And one more thing . . .

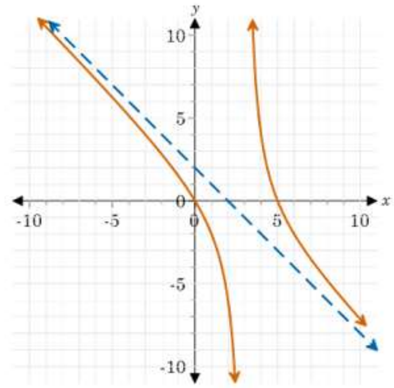
IF the top degree is
exactly one more than
the bottom degree . . .

THEN you have a
Slant Asymptote!



Slant Asymptote:

The equation for this line will be what you get from dividing the top by the bottom, but without the remainder.



Yep, this means polynomial long division.

Slant Asymptote:

The equation for this line will be what you get from dividing the top by the bottom, but without the remainder.

$$f(x) = \frac{x^2 - 3x + 1}{x - 4}$$

$$\begin{array}{r|l} & \quad\; x+1 \\ x-4 & x^2-3x+1 \\ & x^2-4x \;\downarrow \\ \hline & \quad\;\; x+1 \\ & \quad\;\; x-4 \\ \hline & \qquad\; 5 \end{array}$$

Yep, this means polynomial long division.

The slant asymptote for this is the line y=x+1

OPERATIONS with RATIONAL FUNCTIONS:

**Just follow the procedures you've always used for fractions!!

ADD/SUBTRACT: Get a common denominator first.

MULTILIPLY: Top times top & bottom times bottom.

DIVIDE: Multiply by the reciprocal.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.