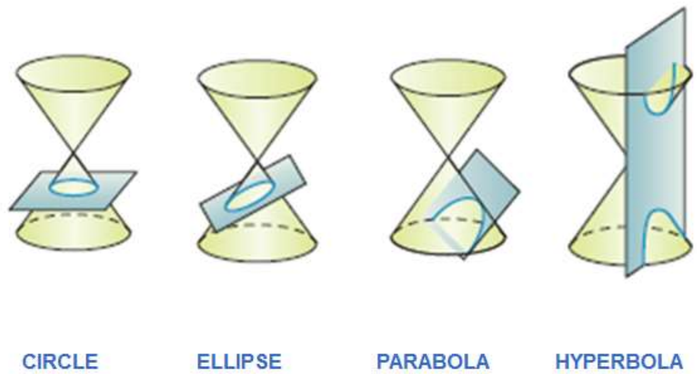
UNIT 6 LESSONS 3-6

PRECALCULUS A

LESSONS:

- Circles
- Ellipses
- Parabolas
- Hyperbolas

CONIC SECTIONS



CONIC SECTIONS

General Equation of a Conic Section

The equation of every conic section can be written in the following form:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, C, D, E, and F are real numbers.

Identifying a Conic Section from Its General Equation

The discriminant of the equation is $B^2 - 4AC$.

- If $B^2 - 4AC > 0$, the conic is a hyperbola.
- If $B^2 - 4AC = 0$, the conic is a parabola.
- If $B^2 - 4AC < 0$ and $A = C$, the conic is a circle.
- If $B^2 - 4AC < 0$ and $A \neq C$, the conic is an ellipse.

But, standard form is more useful . . .

CONIC SECTIONS

Hang in there, we have a lot of details coming!

But don't worry about memorizing all of this . . .

You can use the Key Concepts handout from the review lesson when you do the assessments!!

CIRCLE

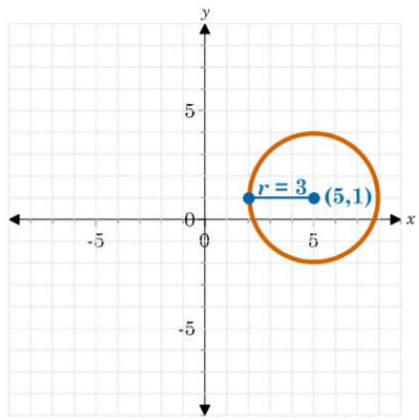
GENERAL FORM - CIRCLE

- $Ax^2 + Cy^2 + Dx + Ey + F = 0$
- $A = C, A \neq 0, C \neq 0$

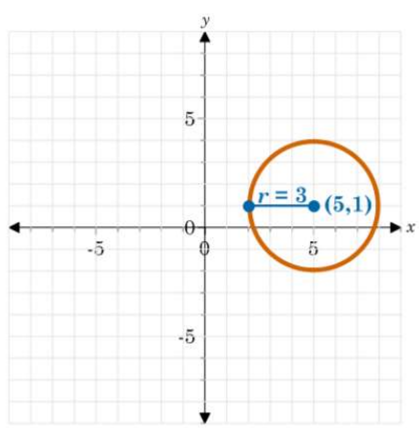
STANDARD FORM - CIRCLE

- $(x - h)^2 + (y - k)^2 = r^2$
- Center at (h, k)
- Radius of r

What is the standard form equation?



What is the standard form equation?



$$(x-5)^2 + (y-1)^2 = 9$$

What is the general form equation?

$$(x-5)^2+(y-1)^2=9$$

The goal is: $\boldsymbol{Ax^2+Cy^2+Dx+Ey+F=0}$

What is the general form equation?

$$(x-5)^2+(y-1)^2=9$$
$$(x-5)(x-5)+(y-1)(y-1)=9$$
(x^2 - 10x + 25) + (y^2 - 2y +1) = 9
x^2 - 10x + y^2 - 2y +26 = 9

Rearrange this to match the pattern:
x^2 + y^2 - 10x - 2y + 17 = 0

The goal is: $\boldsymbol{Ax^2+Cy^2+Dx+Ey+F=0}$

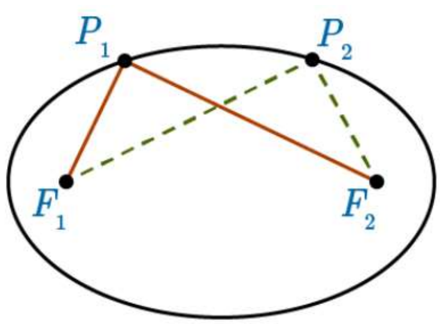
ELLIPSE

- Center
- Foci
- Major axis
- Minor axis
- Vertices

ELLIPSE

- Foci = plural for focus points

Every point on the ellipse has the same combined distance from the foci (focal points).

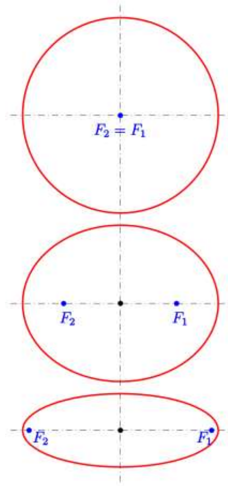


ELLIPSE

- Foci = the focus points

A circle is a special case of ellipse, in which the focal points are the same point, so all circle points are the same distance from the center.

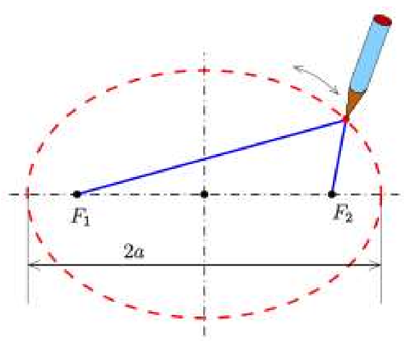
The further apart the foci are, the more elongated the ellipse gets.



ELLIPSE

- Foci = the focus points

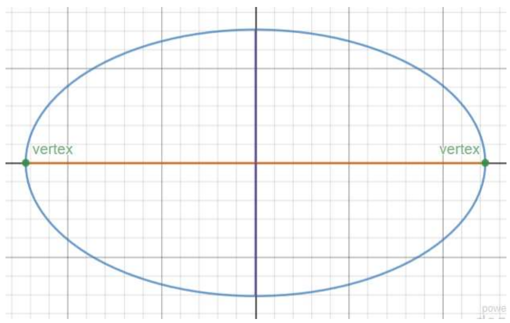
So if you had a string attached to two anchors, and put a pencil holding the string taught, you could draw an ellipse.



ELLIPSE

- Major & Minor axes
- Vertices

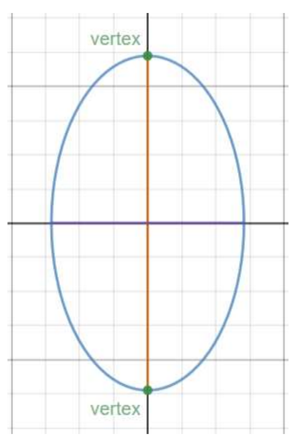
The major axis goes through the center to the furthest points.
The minor axis goes through the center to the closest points.
The vertex points are at the end of the major axis.



ELLIPSE

- Major & Minor axes
- Vertices

An ellipse can be elongated vertically as well.



Major Axis
Minor Axis
Vertices

ELLIPSE

GENERAL FORM - ELLIPSE

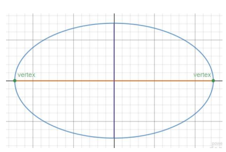
- $Ax^2 + Cy^2 + Dx + Ey + F = 0$
- $A \neq C, A > 0, C > 0$

STANDARD FORM - ELLIPSE

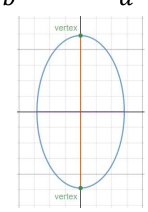
- $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ for horizontal
- $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ for vertical

ELLIPSES

- $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



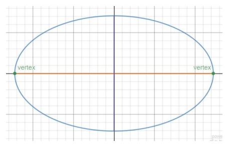
- $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$



- *Notice that the only difference is where a and b go.
- The a is the distance from the center to the edge along the major axis. So, the length of the major axis is 2a.
- The b is the distance from the center to the edge along the minor axis. So, the length of the minor axis is 2b.
- *That means that the larger denominator goes with the variable that matches the direction of the major axis!!

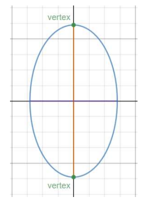
ELLIPSES

- $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



- Vertices at (h+a, k), (h-a, k)
- Foci at (h+c, k), (h-c, k)

- $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$



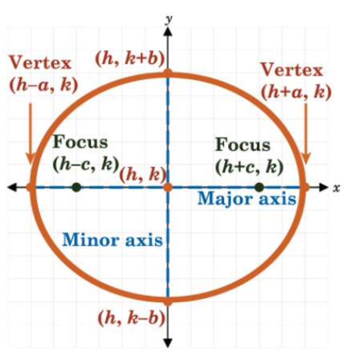
- Vertices at (h, k+a), (h, k-a)
- Foci at (h, k+c), (h, k+c)

For both:

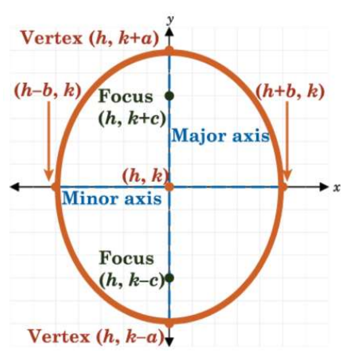
- Major axis length is 2a, Minor axis length is 2b, a>b>0, $c^2 = a^2 - b^2$
- Vertices are at the distance of *a* from the center
- Foci are at the distance of *c* from the center.
- Center at (h, k)

ELLIPSES

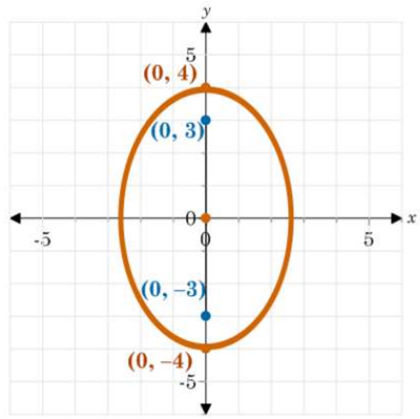
- $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



- $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

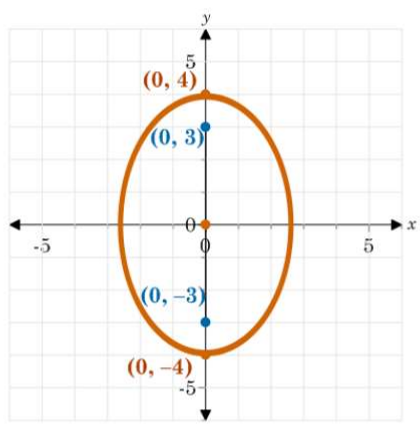


What is the standard form equation?



We need a, b, c, h, k, and which fraction to put the a^2 under.

What is the standard form equation?



Center is (h, k)
So, h=0, k=0.

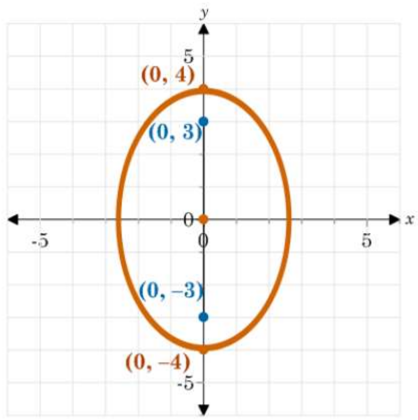
Count center to vertex for a.
So, a=4.

Center to focus point is c.
So, c=3.

What about b?

We need a, b, c, h, k, and which fraction to put the a^2 under.

What is the standard form equation?



Center is (h, k)
So, h=0, k=0.

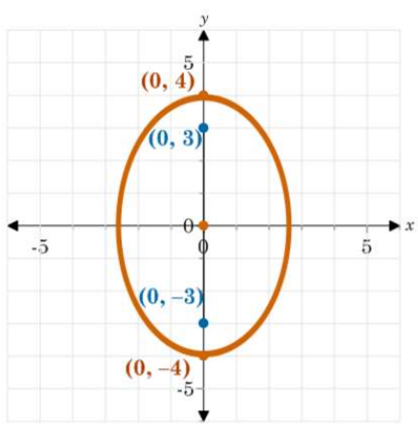
Count center to edge for a.
So, a=4.

Center to focus point is c.
So, c=3.

What about b?
$c^2 = a^2 - b^2$
So, plug in and solve for b.

We need a, b, c, h, k, and which fraction to put the a^2 under.

What is the standard form equation?



h=0
k=0.
a=4.
c=3.
b = $\sqrt{7}$

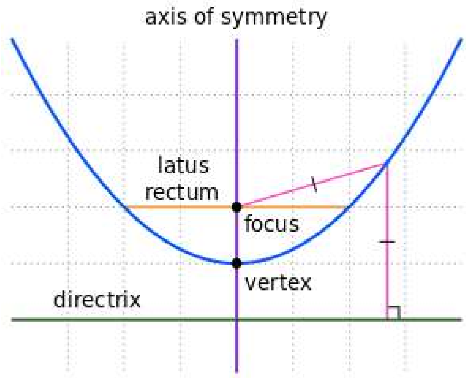
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-0)^2}{\sqrt{7}^2} + \frac{(y-0)^2}{4^2} = 1$$

$$\frac{(x)^2}{7} + \frac{(y)^2}{16} = 1$$

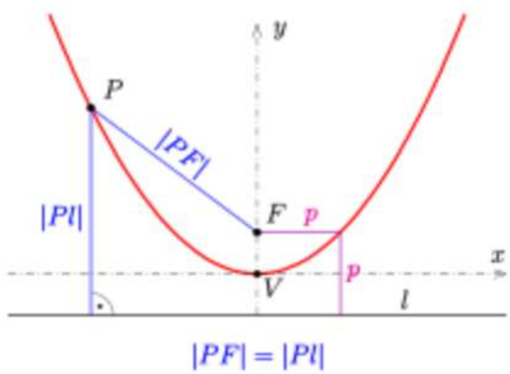
PARABOLA

- Axis of Symmetry
- Focus point
- Directrix line
- Latus Rectum



*Notice that:
- The focus point is the same distance from the vertex as is the directrix line.
- The focus point is always inside the curve, and the directrix line is always outside the curve.

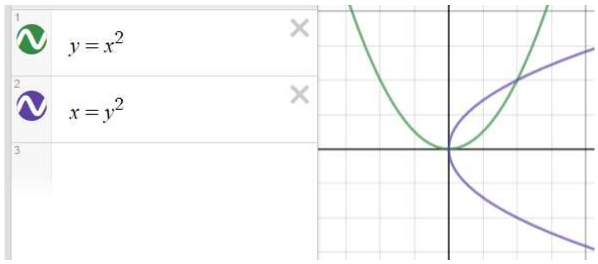
PARABOLA



Every point along a parabola has the same distance from the focus point as from the directrix line.

PARABOLA

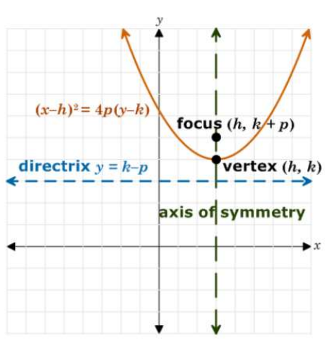
You know it is a parabola if only either the x or the y is squared, but not both!



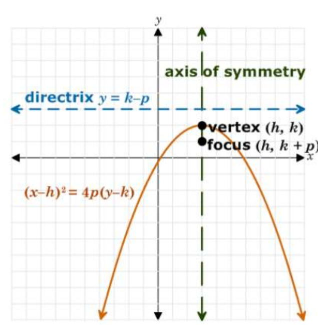
I know, the y^2 parabola is not a function, it doesn't pass the vertical line test.
It's okay, not every useful equation can be called a function.

PARABOLA

Positive x^2 will have a horizontal directrix and a vertical axis of symmetry.

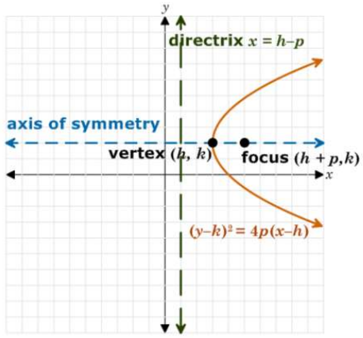


Negative x^2 will have a horizontal directrix and a vertical axis of symmetry.

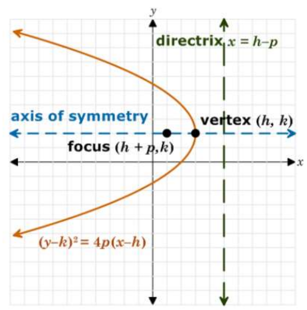


PARABOLA

Positive y^2 will have a vertical directrix and a horizontal axis of symmetry.



Negative y^2 will have a vertical directrix and a horizontal axis of symmetry.



PARABOLAS

GENERAL FORM - PARABOLA

- $Ax^2 + Dx + Ey + F = 0$; For a horizontal directrix
- $Cy^2 + Dx + Ey + F = 0$; For a vertical directrix

STANDARD FORM - PARABOLA

- $(x-h)^2 = 4p(y-k)$; For a horizontal directrix
- Vertex of (h, k); Directrix of y=k-p; Focus of (h, k+p)
- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

PARABOLA

We're used to seeing the parabola equation in vertex form:

$$y = a(x-h)^2 + k$$

Subtract the k, and it starts to look like this new standard form:

$$y - k = a(x-h)^2$$

Replace the a with $\frac{1}{4p}$:

$$y - k = \frac{1}{4p}(x-h)^2$$

Multiply both sides by $4p$:

$$4p(y-k) = (x-h)^2$$

Turn it around to match the format for a horizontal directrix:

$$(x-h)^2 = 4p(y-k)$$

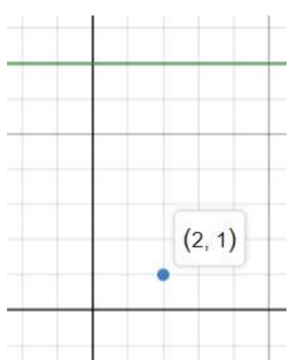
For the vertical directrix, switch the x and y, then switch the h and k.

$$(y-k)^2 = 4p(x-h)$$

What is the standard form equation?

Focus is (2, 1) and the Directrix is y = 7.

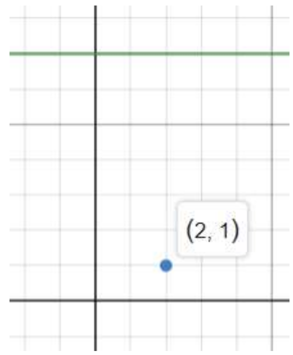
Hint: Sketch this! The value for p, is half the distance from the focus to the directrix. And the vertex (h, k) is at that mid-point.



What is the standard form equation?

Focus is (2, 1) and the Directrix is y = 7.

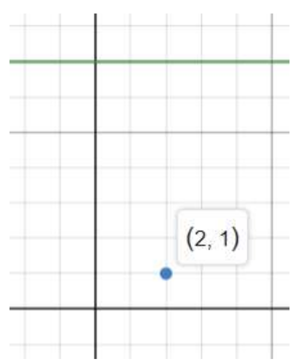
- The distance is 6.
- The focus point is inside the curve, and the directrix is outside the curve, so this will open down.
- The value for p is half this distance, and negative for opening down.
- So, p = -3!!
- The vertex is halfway between the focus and directrix.
- So, the vertex is at (2, 4)



What is the standard form equation?

Focus is (2, 1) and the Directrix is y = 7.

- p = -3
- (h, k) is (2, 4)
- Standard form for a horizontal directrix is: $(x-h)^2 = 4p(y-k)$

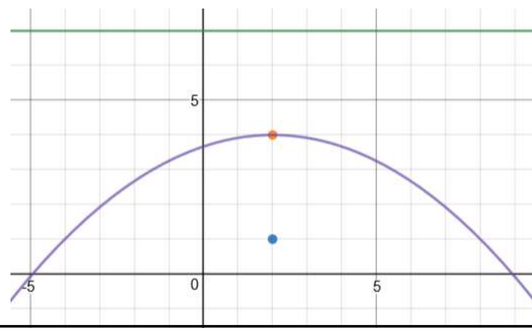


What is the standard form equation?

Focus is (2, 1) and the Directrix is y = 7.

- p = -3
- (h, k) is (2, 4)
- Standard form for a horizontal directrix is: $(x-h)^2 = 4p(y-k)$

$$(x-h)^2 = 4p(y-k)$$
$$(x-2)^2 = 4(-3)(y-4)$$
$$(x-2)^2 = -12(y-4)$$



What is the focus and directrix?

$$(y-5)^2 = -8(x+3)$$

So we need to identify h, k, and p.

Remember, the standard form for a parabola is:

- $(x-h)^2 = 4p(y-k)$; For a horizontal directrix
- Vertex of (h, k); Directrix of y=k-p; Focus of (h, k+p)
- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

What is the focus and directrix?

$$(y-5)^2 = -8(x+3)$$

Remember, h goes with x, and k goes with y!
. . . It's alphabetical ☺

So for this, h = -3, and k = 5.
. . . Remember the pattern is subtract h and k!

And, since the y is squared, this is a vertical directrix.

- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

But what is p?

What is the focus and directrix?

$$(y-5)^2 = -8(x+3)$$

h = -3, and k = 5.

- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

But what is p?

Look at the pattern. There is a 4p in front of the second parentheses.
Look at the equation. There is a -8 in that same spot.

So, . . . 4p = -8

What is the focus and directrix?

$$(y-5)^2 = -8(x+3)$$

h = -3, and k = 5.

- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

p = -2

Now for the focus point coordinates and the directrix line equation!

Look at the pattern . . .

What is the focus and directrix?

$$(y-5)^2 = -8(x+3)$$

h = -3, and k = 5.

- $(y-k)^2 = 4p(x-h)$; For a vertical directrix
- Vertex of (h, k); Directrix of x=h-p; Focus of (h+p, k)

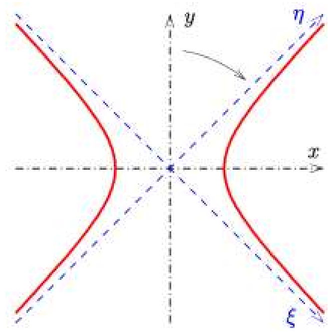
p = -2

*The Focus point is at the coordinates (h+p, k), which is (-5, 5).

*The Directrix line follows the equation x=h-p, which is x = -1.

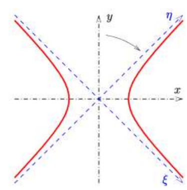
HYPERBOLA

- Branches
- Center
- Vertices
- Axis of Symmetry
- Foci
- Transverse Axis
- Asymptotes
- Conjugate Axis



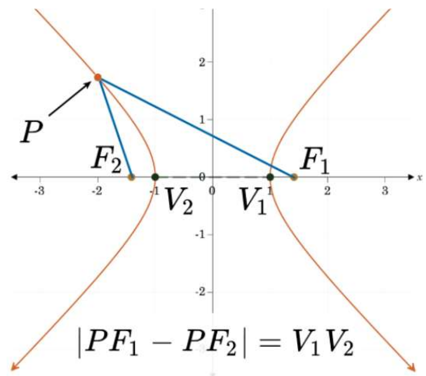
Yes, it's a discontinuous graph.

HYPERBOLA



- Branches – the two parts from the one equation
- Center – the point halfway between the vertices
- Vertices – the turning points of the two branches
- Axis of Symmetry – the "fold" line for the branches
- Foci – the point inside the curve of each branch that helps define the graph
- Transverse Axis – the line segment connecting the vertices . . . Its length is 2a.
- Asymptotes – the "boundary" lines of the branches
- Conjugate Axis – this needs a picture . . . But its length is 2b.

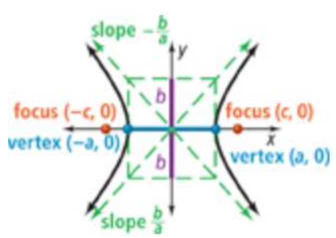
HYPERBOLA

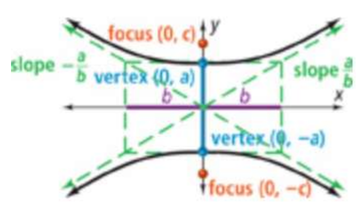


For any point on the hyperbola, the difference (subtract) of its distances from each focus point will be equal to the distance between the vertex points.

HYPERBOLA

- Conjugate Axis – here's the picture . . .





The conjugate axis is the purple line segment!

- Draw a line segment tangent to each vertex to the asymptotes.
- Connect those line segments to make a box.
- The conjugate axis connects these through the center point.
- The length of the transverse axis is 2a.
- The length of the conjugate axis is 2b.
- The distance from center to focus is c.

HYPERBOLA

GENERAL FORM - HYPERBOLA

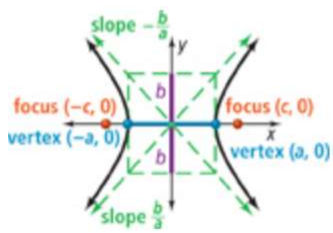
- $Ax^2 - Cy^2 + Dx + Ey + F = 0$ for a horizontal
- -$Ax^2 + Cy^2 + Dx + Ey + F = 0$ for a vertical
- $A \neq C, A > 0, C > 0$

STANDARD FORM - HYPERBOLA

- $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ for a horizontal
- $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ for a vertical

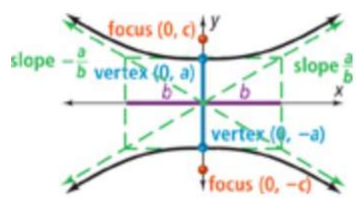
HYPERBOLA

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



- Vertices at (h+a,k), (h-a,k)
- Foci at (h+c,k), (h-c,k)
- Asymptotes at $y = \pm\frac{b}{a}(x-h)+k$
- Center at (h, k)

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



- Vertices at (h,k+a), (h,k-a)
- Foci at (h,k+c), (h,k-c)
- Asymptotes at $y = \pm\frac{a}{b}(x-h)+k$
- Center at (h, k)

- The transverse axis is length 2a
- The conjugate axis is length 2b
- The distance from center to focus is c
- c^2 = a^2 + b^2

CONIC SECTION DETECTIVE

That was a lot!!!

Use the Key Concepts, or your notes, as a reference for doing assessment problems.

With this many possible clues for each type of conic section, you need to think of yourself as a detective to piece these clues together to answer the questions.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.