UNIT 2 LESSON 8 - 9

PRECALCULUS B

LESSONS:

- Inverses of Trig Functions
- Compositions of Trig Functions

And the domain & range for each.

REVIEW: The Vertical Line Function Test

An equation can only be called a function if . . .

?

REVIEW: The Vertical Line Function Test

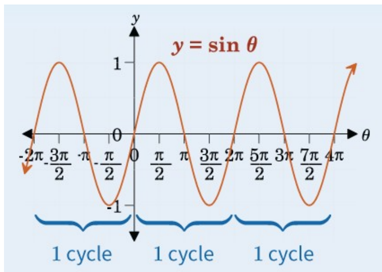
An equation can only be called a function if . . .

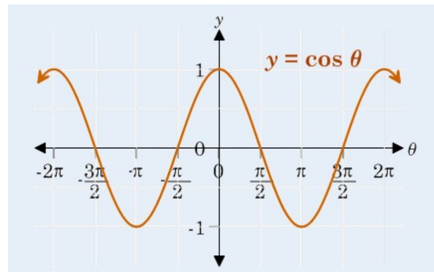
Every x value input yields only 1 y value output.

Check the graph of the equation with a
Vertical Line!

If there is any place where a vertical line crosses the graph more than once, … it fails the test, and cannot be called a function.

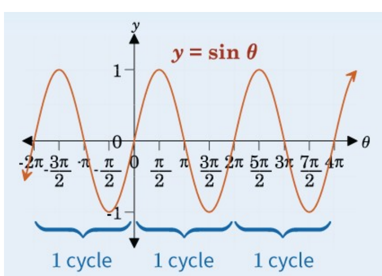
REVIEW: The Vertical Line Function Test

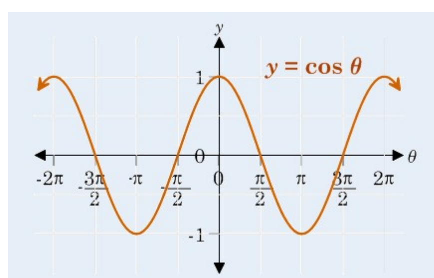




Functions??? Yes or No

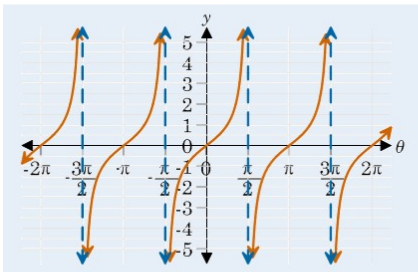
REVIEW: The Vertical Line Function Test

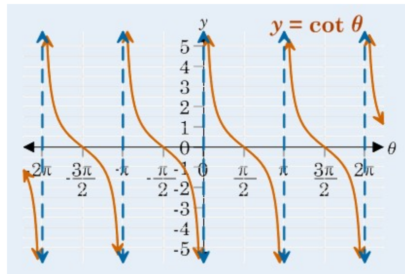




Functions: Yes!!

REVIEW: The Vertical Line Function Test





Functions??? Yes or No

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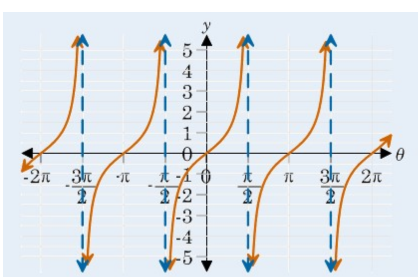
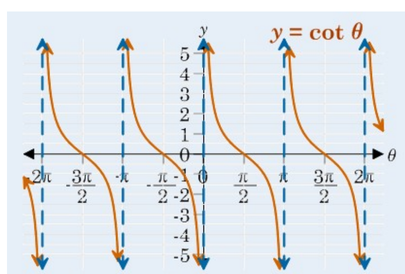
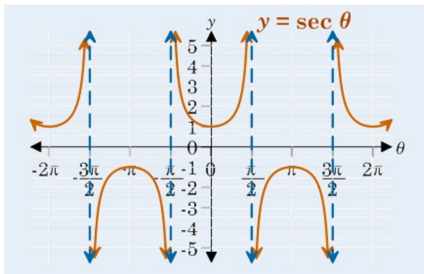


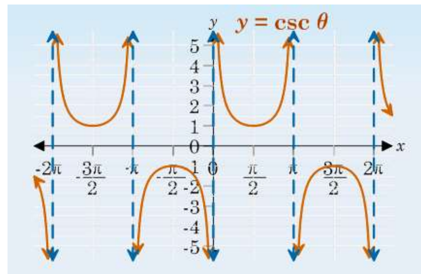


Functions: Yes!!

. . . Remember the asymptotes

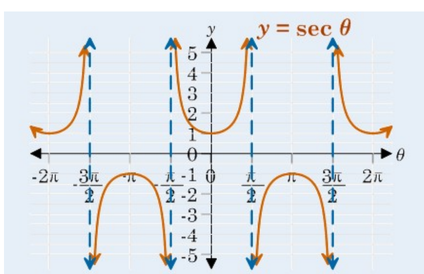
REVIEW: The Vertical Line Function Test

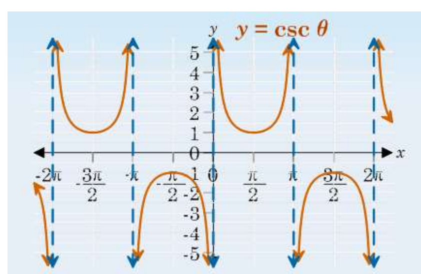




Functions??? Yes or No

REVIEW: The Vertical Line Function Test





Functions: Yes!!

. . . Again, asymptotes

REVIEW: The Horizontal Line Invertible Test

You can tell from the original function if the inverse will also qualify to be called a function, if . . .

?

REVIEW: The Horizontal Line Invertible Test

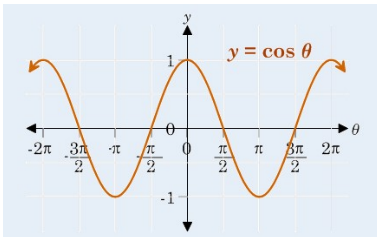
The inverse of a function will also qualify to be called a function, if . . .

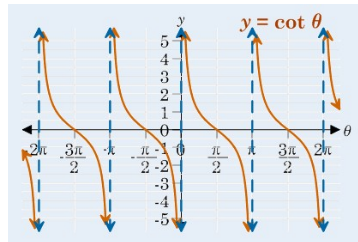
The original function is a one-to-one function.

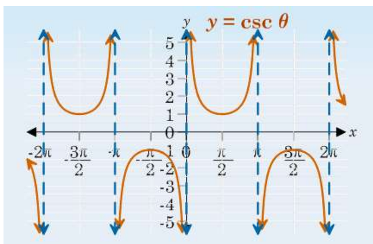
That is, the original function also passes the horizontal line test.

In other words, each y value output occurs only once!

REVIEW: The Horizontal Line Invertible Test

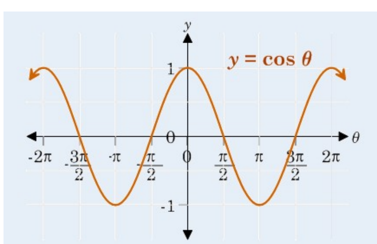


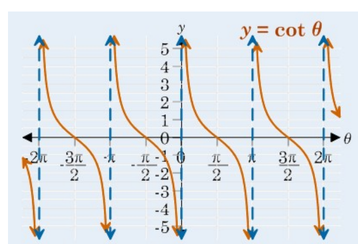


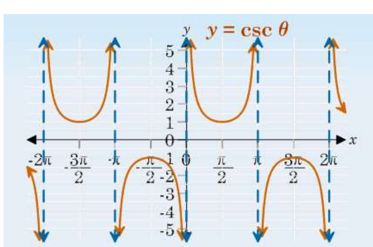


Are these functions one-to-one functions???

REVIEW: The Horizontal Line Invertible Test







No!!
They each fail the horizontal line test.
Which means their inverses can't be called functions.

Inverse Trig Functions???

But inverse functions are useful!!
Being able to "reverse" something is what we do all the time in mathematics.

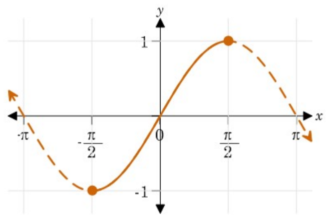
So, if the trig functions are not invertible, that is, their inverses do not qualify as also being called functions, then what do we do???

Inverse Trig Functions!!!

RESTRICT the DOMAIN

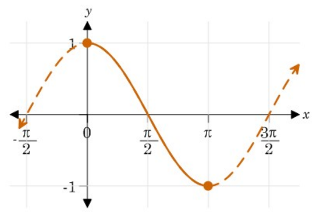
In other words, since trigonometric functions are periodic, we can just use a section of the cycle that does pass the one-to-one test!!

Inverse Trig Functions . . . Restrict the Domain!



Sine

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

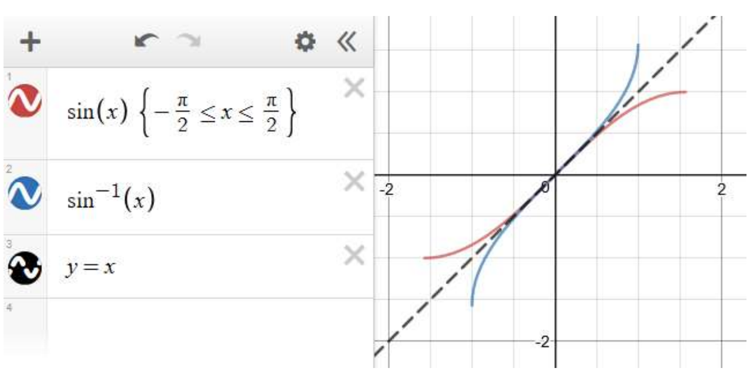


Cosine

$$0 \leq x \leq \frac{\pi}{2}$$

Inverse Trig Functions . . . Restrict the Domain!

And, remember, the graph of any inverse, is the reflection across the diagonal line y=x.



Inverse Trig Functions . . . Restrict the Domain!

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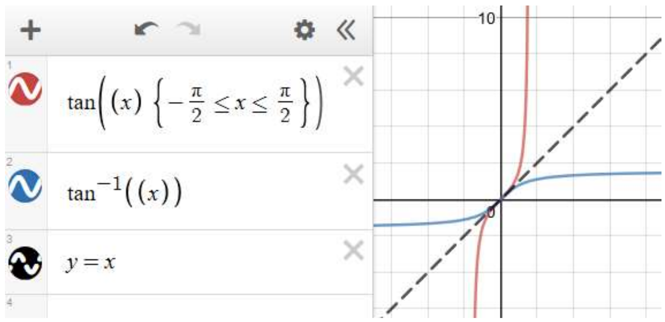

Inverse Trig Functions . . . Restrict the Domain!

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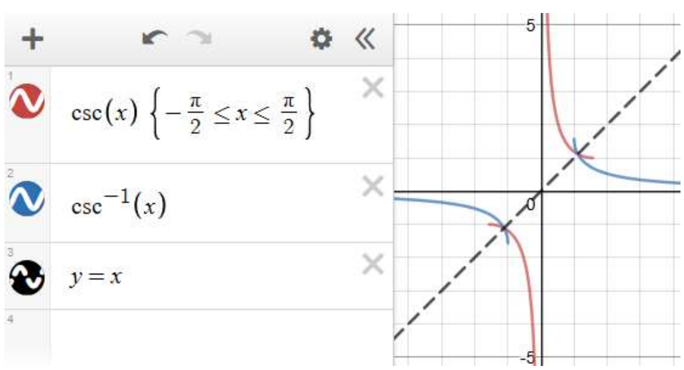


Inverse Trig Functions . . . Restrict the Domain!

Standard Restrictions:

Function	Domain	Range
$\sin x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1,1]$
$\cos x$	$[0, \pi]$	$[-1,1]$
$\tan x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$(-\infty, \infty)$
$\cot x$	$(0, \pi)$	$(-\infty, \infty)$
$\sec x$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$	$(-\infty, -1] \cup [1, \infty)$
$\csc x$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$	$(-\infty, -1] \cup [1, \infty)$

Inverse Trig Functions . . .

Then, the domain of the original, becomes the range of the inverse.

And, the range of the original becomes the domain of the inverse!!!

Inverse Trigonometric Function	Domain	Range
$\sin^{-1} x = \arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x = \arccos x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1} x = \arctan x$	$(-\infty, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cot^{-1} x = \text{arccot} x$	$(-\infty, \infty)$	$(0, \pi)$
$\sec^{-1} x = \text{arcsec} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$\csc^{-1} x = \text{arccsc} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Inverse Trig Functions . . .

Quick note:
An alternate way to write the inverse function for trig functions is to use the prefix "arc".

Inverse Trigonometric Function	Domain	Range
$\sin^{-1} x = \arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1} x = \arccos x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x = \arctan x$	$(-\infty,\infty)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cot^{-1} x = \text{arccot} x$	$(-\infty,\infty)$	$(0,\pi)$
$\sec^{-1} x = \text{arcsec} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$
$\csc^{-1} x = \text{arccsc} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

Inverse Trig Functions . . .

And, . . .
Remember that the answer to an inverse trig function will always be an angle!

$$y = \sin^{-1} x \text{ means } x = \sin y$$

But, . . .
It must be an angle from the restricted domain!

Inverse Trig Functions . . .

TRY:

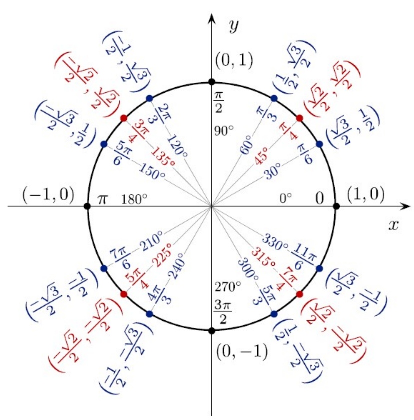
$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

Remember, this is asking what angle, when plugged in to cosine, will give you that fraction as an answer.

Inverse Trig Functions . . .

TRY:

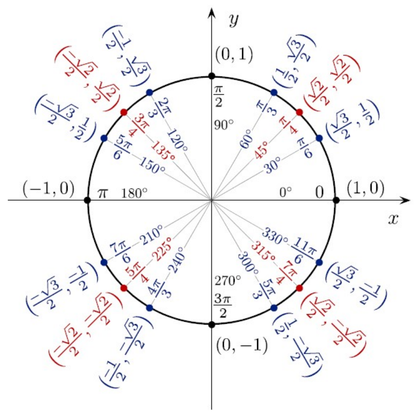
$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



Remember, this is asking what angle, when plugged in to cosine, will give you that fraction as an answer. Cosine is the x-coordinate.

TRY:

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

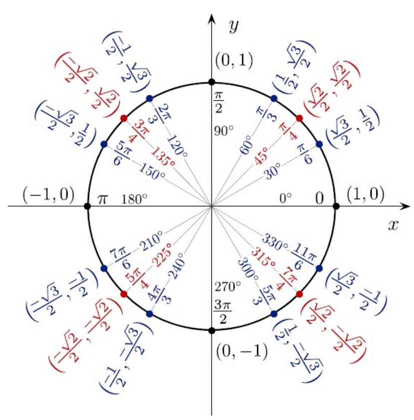


There are four angles where cosine is √2/2.
Only two are positive – Quadrants I & IV.
The domain for cosine is restricted to [0, π] – or, Quadrants I & II.

What is the angle that answers this inverse function??

TRY:

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



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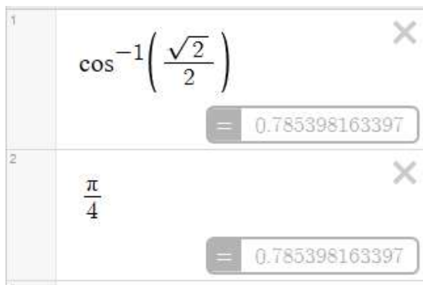
The angle must be the one in Quadrant I.
This occurs at π/4.
So this is the answer for the inverse function!

TRY:

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

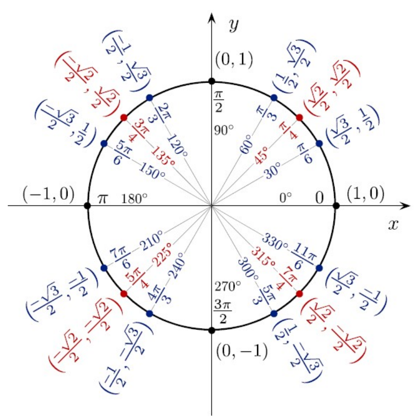
The angle must be the one in Quadrant I.
This occurs at π/4.
So this is the answer for the inverse function!

Desmos will give you the approximate decimal value.



TRY:

$$\csc^{-1}(-2)$$

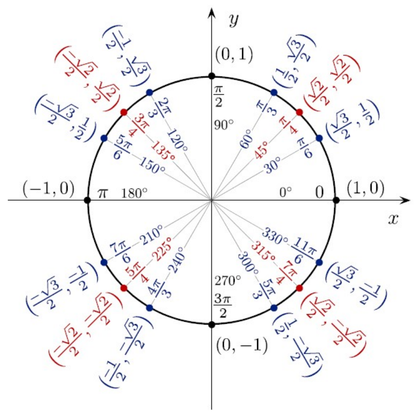


Remember, on the unit circle,
Each point (x, y) corresponds to (cos, sin).
And tangent is sin/cos, which becomes y/x.

So, which of these functions is csc the reciprocal of??

TRY:

$\csc^{-1}(-2)$

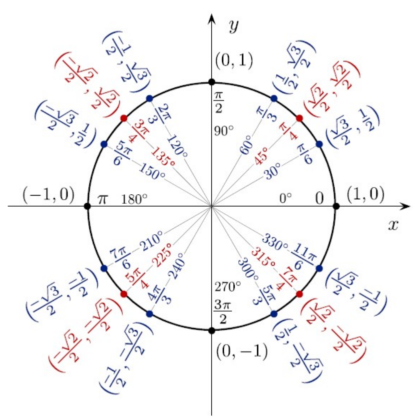


Yes, csc is the reciprocal of sin!
So, this is the same as looking for the angle that makes sin = -1/2.

Where do you see y-coordinates of -1/2??

TRY:

$\csc^{-1}(-2)$



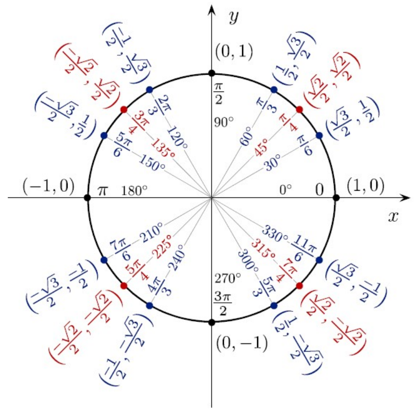
There are two places where sine = -1/2, in Quadrants III & IV.
Which one is in the restricted domain for csc??

That is, it must be between −π/2 and +π/2, but not at 0

What is the angle that answers this inverse function??

TRY:

$\csc^{-1}(-2)$

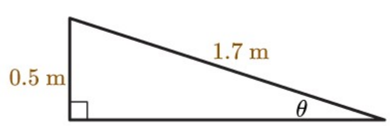


This is at 11π/6.

Or, going clockwise, it could also be said to be at −π/6.

Now, to apply this to real life . . .

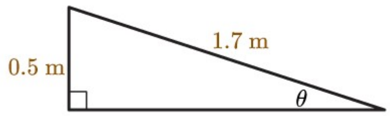
When you know the length of two sides of a right triangle,
and you need to know the measurement of an angle,
using the appropriate SOH-CAH-TOA ratio
and it's inverse, you can calculate the angle measurement.



Which trig ratio would you use for this theta angle??

Now, to apply this to real life . . .

Yes, opposite and hypotenuse would go with sine!



So, sin θ = opp/hyp = 0.5/1.7

Which means, arcsin (0.5/1.7) = θ

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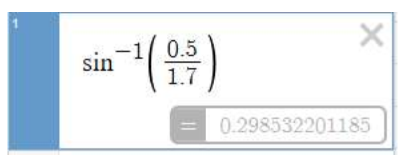
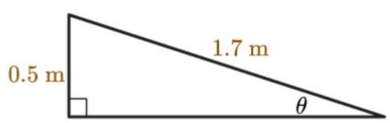
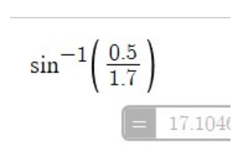
arcsin (0.5/1.7) = θ

Θ is ~ 0.3 radians





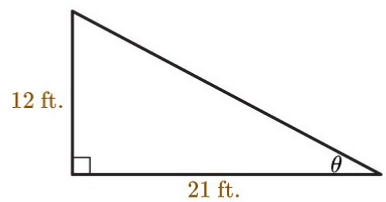
Or, about 17 degrees.



Remember, Desmos defaults to radians.
If you need your answer in degrees,
use the wrench icon to change it.

Find the angle . . .

First, which trig ratio to use?



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If you need your answer in degrees,
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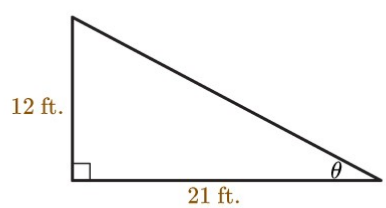
Find the angle . . .

Yes, tangent is opposite/adjacent!



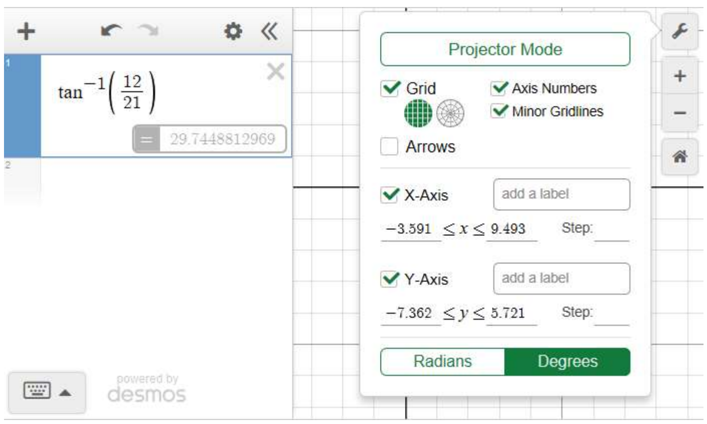
$\tan \theta$ = opp/adj = 12/21

arctan (12/21) = θ

Find the angle . . .

arctan (12/21) = θ

Θ is ~ 30 degrees



Next: Compositions of Trig Functions

f(g(x)) “f of g of x”

Remember, this means whatever you get from plugging x into the g function, is what you then plug into the f function.

Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

Find f(g(π)), which would be cos(sin(π))

. . . Note that this means use π for x.

Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

Find f(g(π))

1st: g(π) = sin(π) = 0

2nd: f(g(π)) = f(0) = cos(0) = 1

So, f(g(π)) = cos(sin(π))= 1 radian

Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

But what about g(f(π))??

Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

But what about g(f(π))??

1st: $f(\pi) = \cos(\pi) = -1$

2nd: $g(f(\pi)) = g(-1) = \sin(-1) \approx -0.84$ rad

So, $g(f(\pi)) = \sin(\cos(\pi)) \approx -0.84$ rad

Not the same, so they are not inverses.

Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

What is the domain and range of g(f(x))??
That is, of sin(cos(x)) . . .

Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

What is the domain and range of g(f(x))??
That is, of sin(cos(x))

1st: What is the domain of the first function used?
That is, what is the domain of cos x?

2nd: What is the range for the output of the first function?

Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

What is the domain and range of g(f(x))??
That is, of sin(cos(x))

1st: The domain of cos x is all real numbers are okay to use for input.

2nd: The range for cos x is all numbers between -1 and 1 are what you get as output.

Domain & Range
of Compositions of Trig Functions

1st: The domain of cos x is all real numbers are okay to use for input.

2nd: The range for cos x is all numbers between -1 and 1 are what you get as output.

***That means you can only use numbers between -1 and 1 as input to the next function of the composite!!!

Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

What is the domain and range of g(f(x))??
That is, of sin(cos(x))

3rd: Check what you get in the second function if your input is restricted to the range output of the first function.

For this, you can only input -1 to 1 into sin x.

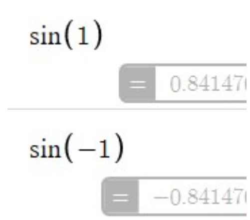
BTW, we are still working in radians.

Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,

What is the domain and range of g(f(x))??
That is, of sin(cos(x))

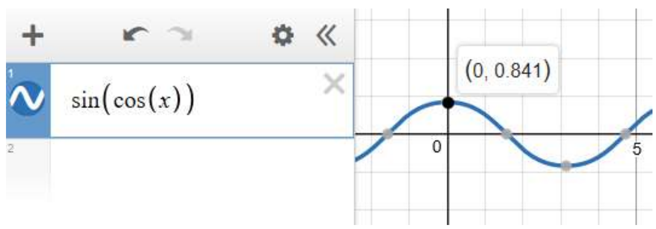
These are what you get for sine at the limits of what we can input after doing cosine.



Domain & Range
of Compositions of Trig Functions

If f(x) = cos x, and g(x) = sin x,
What is the domain and range of g(f(x))??
That is, of sin(cos(x))??

Domain: x is all real numbers
Range: -0.84 ≤ y ≤ 0.84



Domain & Range
of Compositions of Trig Functions

A handy reference for getting started:

Function	Domain	Range	Inverse	Domain	Range
sin	all real numbers	$-1 \le y \le 1$	arcsin	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cos	all real numbers	$-1 \le y \le 1$	arccos	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
tan	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	all real numbers	arctan	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
csc	all real numbers, $x \ne n\pi$, where n is an integer	$-1 > y$ $y > 1$	arccsc	$-1 > x$ $x > 1$	$0 < y < \pi$
sec	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	$-1 > y$ $y > 1$	arcsec	$-1 > x$ $x > 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cot	all real numbers, $x \ne n\pi$ where n is an integer	all real numbers	arccot	all real numbers	$0 < y < \pi$

Domain & Range
of Compositions of Trig Functions

Yes, you can compose regular and inverse functions!

Function	Domain	Range	Inverse	Domain	Range
sin	all real numbers	$-1 \le y \le 1$	arcsin	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cos	all real numbers	$-1 \le y \le 1$	arccos	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
tan	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	all real numbers	arctan	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
csc	all real numbers, $x \ne n\pi$, where n is an integer	$-1 > y$ $y > 1$	arccsc	$-1 > x$ $x > 1$	$0 < y < \pi$
sec	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	$-1 > y$ $y > 1$	arcsec	$-1 > x$ $x > 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cot	all real numbers, $x \ne n\pi$ where n is an integer	all real numbers	arccot	all real numbers	$0 < y < \pi$

Domain & Range
of Compositions of Trig Functions

Yes, you can compose regular and inverse functions!

Look at: cos(arctan(x))

*Remember that an inverse function equals an angle!!

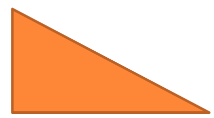
So, arctan(x) means the angle
that gives you a tangent of x!

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

1st for an inverse function: Draw the right triangle for the ratio that goes with the inverse function.

Since, arctan x means the angle that gives you a tangent of x,



And since tangent = opposite/adjacent,

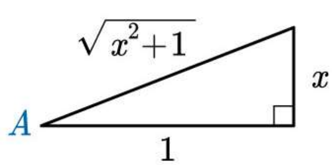
Then, tan θ = x means the opposite side is x and the adjacent side is 1.

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

2nd: Use the Pythagorean Theorem to find the other side.

This is the triangle for tan θ = x where the opposite is x and the adjacent is 1. Or you can use A for θ.

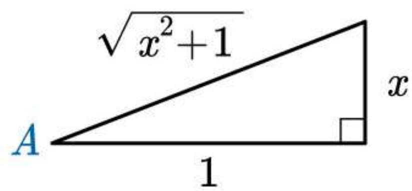


Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

3rd: Use this triangle for the second function.

cos(arctan(x)) = cos A = ??

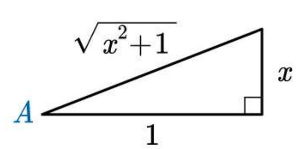


Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

cos(arctan(x)) = cos A = adj/hyp

$$\cos(\arctan x) = \frac{1}{\sqrt{x^2+1}}$$



Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

But, what are the domain and range??

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

What are the domain and range??
The domain of the composition is the domain of the inside function.

Function	Domain	Range	Inverse	Domain	Range
sin	all real numbers	$-1 \le y \le 1$	arcsin	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cos	all real numbers	$-1 \le y \le 1$	arccos	$-1 \le x \le 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
tan	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	all real numbers	arctan	all real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
csc	all real numbers, $x \ne n\pi$, where n is an integer	$-1 > y$ $y > 1$	arccsc	$-1 > x$ $x > 1$	$0 < y < \pi$
sec	all real numbers, $x \ne \frac{\pi}{2} + n\pi$, where n is an integer	$-1 > y$ $y > 1$	arcsec	$-1 > x$ $x > 1$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
cot	all real numbers, $x \ne n\pi$ where n is an integer	all real numbers	arccot	all real numbers	$0 < y < \pi$

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

What are the domain and range??
The domain of arctan is x is all real numbers.

The range of the composite is what you get from using the range of the first function as the domain of the second function.

So, what is the range of arctan??

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

The domain of the composite is x is all real numbers.

What is the range of the composite?

The range of arctan is $-\pi/2$ to $\pi/2$

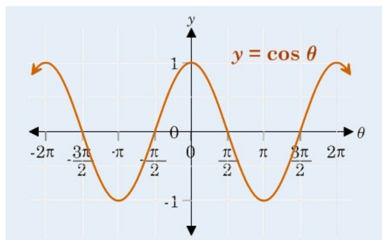
So, what do you get for cosine when you only input between these values?

Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

The domain of the composite is x is all real numbers.

What do you get for cosine between $-\pi/2$ and $\pi/2$?

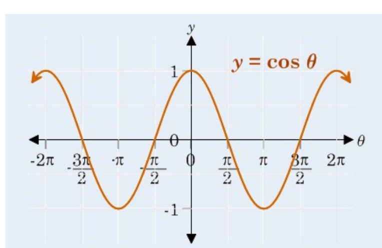


Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

The domain of the composite is x is all real numbers.

Between $-\pi/2$ and $\pi/2$ for cosine, you get $0 \le y \le 1$



Domain & Range
of Compositions of Trig Functions

Look at: cos(arctan(x))

The domain of the composite is x is all real numbers.
And the range of the composite is $0 < y \leq 1$.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!