Trigonometric Identities and Applications Key Concepts

Trigonometric Identities Lesson

Fundamental Trigonometric Identities

The fundamental trigonometric identities can be used to prove, verify, or solve many trigonometric expressions or equations. These identities are the following:

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\sin\theta = \dfrac{1}{\csc\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Sum and Difference Formulas Lesson

Sum and Difference Formulas

1. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
3. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
4. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
5. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
6. $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double-Angle Formulas Lesson

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Half-Angle Formulas

$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$ for $\frac{\theta}{2}$ in Quadrant I or II

$\sin\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{2}}$ for $\frac{\theta}{2}$ in Quadrant III or IV

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$ for $\frac{\theta}{2}$ in Quadrant I or IV

$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}}$ for $\frac{\theta}{2}$ in Quadrant II or III

$$\tan\frac{\theta}{2}=\frac{\sin\theta}{1+\cos\theta}$$

$$\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}$$

Law of Sines Lesson

Law of Sines

If *A*, *B*, and *C* are the measures of the angles of a triangle and *a*, *b*, and *c* are the lengths of the corresponding sides, then $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin\ C}{c}$.

The Law of Sines is true for all triangles.

The Ambiguous Case

The Law of Sines works when given ASA or AAS triangles. If you are given an SSA triangle, you need to determine whether there are zero, one, or two possible solutions. If *A* is acute and $a<h$, no triangle exists. If *A* is acute and *a* = *h* or $a>b$, then one triangle exists. If angle *A* is acute and $h<a<b$, there are two possible triangles. If *A* is obtuse and $a\leq b$, no possible triangle exists. If *A* is obtuse and $a>b$, one triangle exists.

Law of Cosines Lesson

Law of Cosines

If *A*, *B*, and *C* are the measures of the angles of a triangle and *a*, *b*, and *c* are the lengths of the corresponding sides, then the following applies:

$$a^2=b^2+c^2-2bc\cos A$$

$$b^2=a^2+c^2-2ac\cos B$$

$$c^2=a^2+b^2-2ab\cos C$$

The Law of Cosines is true for all triangles.

Alternative Form of the Law of Cosines

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Trigonometric Equations Lesson

Algebraic Strategies for Solving Trigonometric Equations

- factoring
- squaring
- quadratic formula