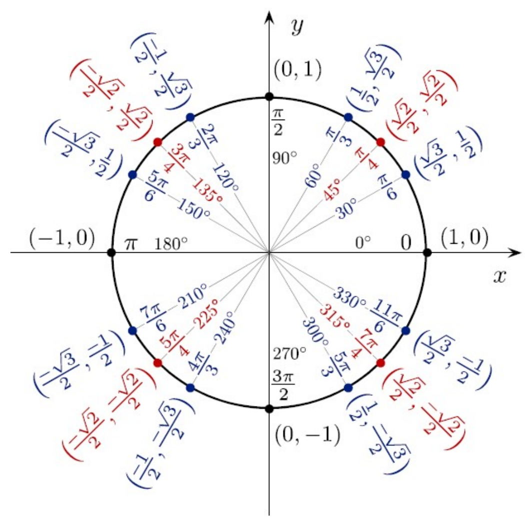
UNIT 3 LESSON 3

PRECALCULUS B

LESSON 3:

- Double-Angle Formulas
  - Double-Angle
  - Power-Reducing
  - Half-Angle

We will need the Unit Circle:



DOUBLE-ANGLE FORMULAS

Double-Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Notice that cosine as 3 versions!

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Since cosine as three versions, use whichever version is most helpful to the problem you are working on!

DOUBLE-ANGLE FORMULAS

And, feel free to rearrange any formula or identity to isolate for a term you need a substitution for!

Here's a cosine double angle formula rearranged . . .

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta - 1 = -2\sin^2\theta$$

$$\frac{1-\cos 2\theta}{2} = \sin^2\theta$$

Now you have another substitution for $\sin^2(\theta)$

POWER-REDUCING FORMULAS

Power-Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

These formulas give you a substitution without squares for a squared function!

That could come in handy.

HALF-ANGLE FORMULAS

When solving for an angle not on the Unit Circle, instead of adding or subtracting to get it, sometimes you can cut a known angle in half.

Here is one of the half-angle formulas:

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

What if we used this for finding sin(15) as sin(30/2),

instead of earlier as sin(45-30) . . .

HALF-ANGLE FORMULAS

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

sin(15) = sin(30/2) = √1-cos(30)/2

≈ 0.2588

Same answer we got from using the sum formula!

HALF-ANGLE FORMULAS

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} \quad \text{for } \frac{\theta}{2} \text{ in Quadrant I or II}$$

$$\sin\frac{\theta}{2} = -\sqrt{\frac{1-\cos\theta}{2}} \quad \text{for } \frac{\theta}{2} \text{ in Quadrant III or IV}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} \quad \text{for } \frac{\theta}{2} \text{ in Quadrant I or IV}$$

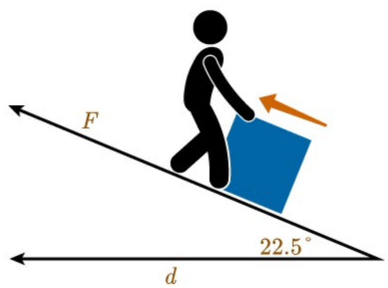
$$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} \quad \text{for } \frac{\theta}{2} \text{ in Quadrant II or III}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

Example:

The work done by a force is given by the equation $W = Fd\cos\theta$, where F is the magnitude of the force, d is the distance traveled by the object, and θ is the angle between the direction of the force and the direction of movement. How much work is done by a man dragging a box with 25 N of force at a 22.5° angle for 2 m? Find the exact value.



W = Fd cosθ

W = (25)(2)cos(22.5)

How do we get 22.5° from the unit circle?

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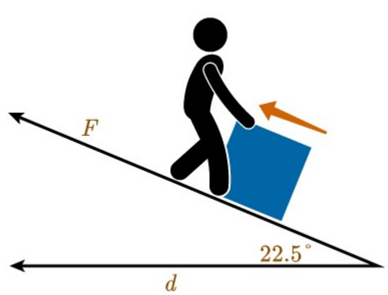

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An angle of 22.5° is half of 45°!

So, we need the cosine half angle formula.

Example:

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So, we need the cosine half angle formula.

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} \quad \text{for } \frac{\theta}{2} \text{ in Quadrant I or IV}$$

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Example:

$$W = Fd\cos\theta$$

$$W = (25)(2)\cos 22.5°$$

$$W = 50\cos\frac{45°}{2}$$

$$W = 50\sqrt{\frac{1+\cos 45°}{2}}$$

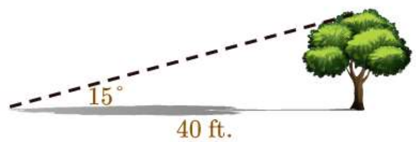
$$W = 50\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$

$$W = 50\sqrt{\frac{2+\sqrt{2}}{4}}$$

$$W = 25\sqrt{2+\sqrt{2}}$$

Example:

A tree casts a shadow 40 ft. long when the sun is 15° above the horizon. How tall is the tree? Find the exact value.



So the tree is opposite the angle, and the shadow is adjacent to the angle …

tan θ = opp/adj

tan 15° = h/40

40 tan 15° = h

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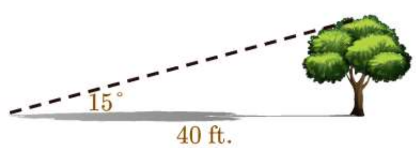

tan θ = opp/adj

tan 15° = h/40

40 tan 15° = h

How do we get 15° from the unit circle?

Half of 30° is 15°!

Example:

$$h = 40\tan\frac{30°}{2}$$

$$h = 40\frac{\sin 30°}{1+\cos 30°}$$

$$h = 40\frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

$$h = 40\frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$$

$$h = 40\left(\frac{1}{2+\sqrt{3}}\right)$$

You will also use these formulas in the substitution steps of verifying identities.

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Power-Reducing Formulas

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Example:

Verify that $\cos^2\frac{\theta}{2}=\frac{\tan\theta+\sin\theta}{2\tan\theta}$.

*Remember, to verify an identity, substitute and simplify to the left side until it matches the right side.

Possible starting strategies:

- The left is degree 2 and the right is degree 1, so maybe use a power reducing formula.
- The left has a half angle, so probably use a half angle formula.

Example:

Verify that $\cos^2\frac{\theta}{2}=\frac{\tan\theta+\sin\theta}{2\tan\theta}$.

$\cos^2\frac{\theta}{2}=\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2$ Apply the half-angle formula for cosine.

$\cos^2\frac{\theta}{2}=\frac{1+\cos\theta}{2}$ Simplify.

$\cos^2\frac{\theta}{2}=\frac{1+\cos\theta}{2}\left(\frac{\tan\theta}{\tan\theta}\right)$ Multiply by a factor of 1.

$\cos^2\frac{\theta}{2}=\frac{\tan\theta+\tan\theta\cos\theta}{2\tan\theta}$ Distribute.

$\cos^2\frac{\theta}{2}=\frac{\tan\theta+\frac{\sin\theta}{\cos\theta}\cos\theta}{2\tan\theta}$ Apply the reciprocal identity.

$\cos^2\frac{\theta}{2}=\frac{\tan\theta+\sin\theta}{2\tan\theta}$ Simplify.

Train Your Brain:

Working through the steps in the lesson examples
and figuring out what is done in each step
will help train your brain to see possibilities
for putting together the puzzles of new problems!

YES, you may need to try
more than one strategy
to find a way to make it work!!

It is like doing a puzzle or a maze . . .

Be patient with the process
and take your time!

Questions??

Review the Key Terms and Key Concepts
documents for this unit.

Look up the topic at khanacademy.org and
virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day & time.

Reserve a time for a call with me at
jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over
problems together!