UNIT 3 LESSON 5
Law of Cosines

PRECALCULUS B

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

**NOTE: This involves all three sides and one angle.
So if you know three of these four numbers,
you can solve for the missing one!**

5th Case: Given 1 Angle & 2 Sides - SAS

Start by labeling the triangle, and setting up the corresponding Law of Cosines:

Let $a = 80$, $b = 120$, and $C = 133°$.

$c^2 = a^2 + b^2 - 2ab\cos C$

5th Case: Given 1 Angle & 2 Sides - SAS

Use the Law of Cosines to solve for the third side.

TRY IT: Plug in, and calculate the length of side c.

Let $a = 80$, $b = 120$, and $C = 133°$.

$c^2 = a^2 + b^2 - 2ab\cos C$

5th Case: Given 1 Angle & 2 Sides - SAS

Use the Law of Cosines to solve for the third side.

TRY IT: Plug in, and calculate the length of side c.

Let $a = 80$, $b = 120$, and $C = 133°$.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (80)^2 + (120)^2 - 2(80)(120)\cos 133°$$

5th Case: Given 1 Angle & 2 Sides - SAS

Use the Law of Cosines to solve for the third side.

TRY IT: Plug in, and calculate the length of side c.

Let $a = 80$, $b = 120$, and $C = 133°$.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (80)^2 + (120)^2 - 2(80)(120)\cos 133°$$

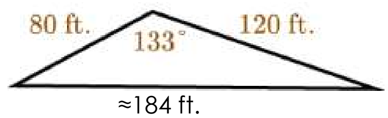
$$c^2 = 6,400 + 14,400 - (-13,094.37)$$

$$c^2 = 33,894.37$$

$$c \approx 184$$

5th Case: Given 1 Angle & 2 Sides - SAS

Now you have angle C and side c to start the Law of Sines to solve for a second angle!

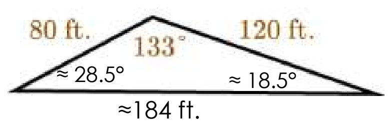


Try It!

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

C ≈ 28.5°

5th Case: Given 1 Angle & 2 Sides - SAS



Once you have a second angle, you could use Law of Sines again to get the third angle, but it would be quicker to just subtract the two angles you now have from 180°.

180 – 133 – 28.5 = 18.5

5th Case: Given 1 Angle & 2 Sides - SAS

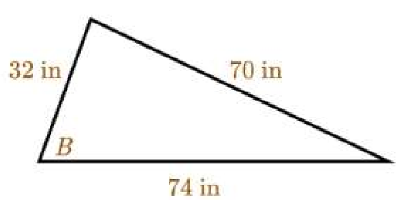
Steps for Solving a SAS:

- ❑ Use Law of Cosines to solve for the third side.
- ❑ Use that third side with the known angle to set up a Law of Sines proportion to solve for a second angle.
- ❑ Use the two known angles to subtract from 180 to get the third angle.

The triangle is SOLVED!

6th Case: Given 3 Sides - SSS

This is another case for Law of Cosines!



Again, you don't have a known angle and opposite side pair to start the Law of Sines . . .

But the Law of Cosines is set up to solve for a missing side, not a missing angle!

Law of Cosines v.2

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

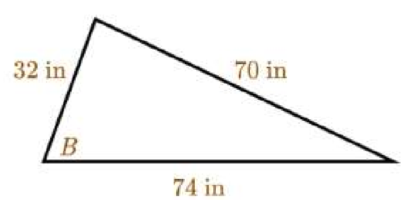
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Another Version of the same New Tool ☺

NOTE: This still uses three sides and one angle.
BUT, this version is rearranged to solve for the angle when all three sides are known!

6th Case: Given 3 Sides - SSS

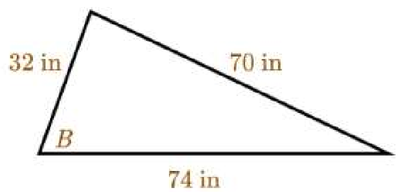
Again, start by labeling the triangle, and setting up the corresponding Law of Cosines:



Let $a = 32$, $b = 70$, and $c = 74$.

6th Case: Given 3 Sides - SSS

Since you know all three sides, to get the first angle, set up the corresponding alternate version of Law of Cosines:

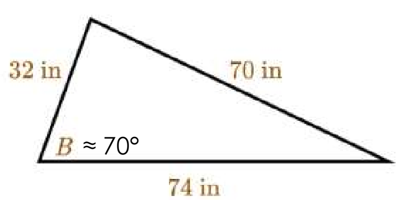


Let $a = 32$, $b = 70$, and $c = 74$.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

6th Case: Given 3 Sides - SSS

Solve for the first angle:



Let $a = 32$, $b = 70$, and $c = 74$.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{32^2 + 74^2 - 70^2}{2(32)(74)}$$

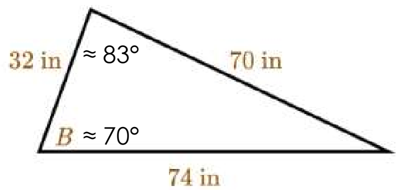
$$\cos B = \frac{1{,}024 + 5{,}476 - 4{,}900}{4{,}736}$$

$$\cos B \approx 0.3378$$

$$B \approx 70^\circ$$

6^{th} Case: Given 3 Sides - SSS

Now you have angle B and side b to start the Law of Sines to solve for a second angle!

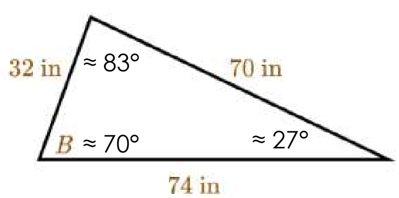


Try It:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

C ≈ 83°

6^{th} Case: Given 3 Sides - SSS



Once you have a second angle, subtract the two angles you now have from 180° to get angle A.

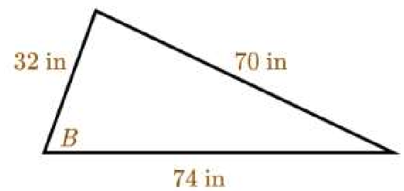
180 – 70 – 83 = 27

6th Case: Given 3 Sides - SSS

Steps for Solving a SSS:

- Use the alternate version of Law of Cosines to solve for one angle.
- Use that angle with its opposite side to set up Law of Sines to solve for a second angle.
- Use the two angles to subtract from 180 to get the third angle.

The triangle is SOLVED!



Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day and time.

*Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!