Parametric Functions Key Concepts

Plane Curves and Parametric Functions Lesson

Plane Curve

If *f* and *g* are continuous functions of *t* on an interval *I*, the set of ordered pairs $\left(f(t), g(t)\right)$ is a plane curve *C*.

The equations $x = f(t)$ and $y = g(t)$ are the parametric equations for the plane curve *C*.

The variable *t* is called the parameter.

Plotting Points and Eliminating the Parameter Lesson

Graphing a Plane Curve by Plotting Points

1. Evaluate $f(t)$ and $g(t)$ at a variety of values of *t* to calculate *x* and *y.*
2. Plot points (x, y) and connect them.
3. Represent the orientation of the curve by placing arrows along the curve as *t* increases.

Solving for *t*

1. Solve one of the parametric equations for *t*.
2. Substitute the expression for *t* into the other parametric equation.
3. Rewrite the equation in a form with which you are familiar.
4. Plot points to determine the orientation.

Solving for *x* or *y*

1. Use substitution to write one of the variables in terms of the other variable.
2. Rewrite the equation in a form with which you are familiar.
3. Plot points to determine the orientation.

Using Trigonometric Identities Lesson

Rewriting Parametric Equations Using Trigonometric Identities

1. Determine which trigonometric identity can be used.
2. Manipulate the equations, if needed.
3. Substitute the expressions with *x* and *y* into the trigonometric identity.
4. Rewrite in a form with which you are familiar.

Finding Parametric Equations Lesson

Writing Parametric Equations

One plane curve can be represented by infinitely many parametric equations.

One set of parametric equations that represent the same path as the function $y = f(x)$ is $x(t) = t$ and $y(t) = f(t)$, where *t* is in the domain of *f.*

Additional sets of parametric equations for $f(x)$ can be found by using different functions for $x(t)$, as long as the function allows *x* to take on all values in the domain of $f(x)$.

Polar and Parametric Equations Lesson

Polar to Parametric

To convert a polar function $r(\theta)$ to parametric equations, use $x = r\cos\theta$ and $y = r\sin\theta$, where θ is now the parameter.

Parametric to Polar

To convert parametric equations $x(t)$ and $y(t)$ to a polar function $r(\theta)$, use the formulas $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ and $r(\theta) = \sqrt{x(\theta)^2 + y(\theta)^2}$.

Modeling Linear and Projectile Motion Lesson

Parametric Equations Representing Linear Motion

Linear motion can be represented by the parametric equations $x = (v\cos\theta)t + x_0$ and $y = (v\sin\theta)t + y_0$, where *v* is the object's velocity, θ is the angle the object makes with the *x*-axis (or line parallel to the *x*-axis), and (x_0, y_0) is the starting point.

Parametric Equations Representing Projectile Motion

Projectile motion can be represented by the parametric equations $x = (v\cos\theta)t + x_0$ and $y = -\frac{1}{2}gt^2 + (v\sin\theta)t + y_0$, where *v* is the object's initial velocity, *θ* is the angle the object makes with the *x*-axis (or line parallel to the *x*-axis), (x_0, y_0) is the starting point, and *g* is the acceleration due to gravity.