Looking Ahead to Calculus Key Concepts

Limit of a Function Lesson

Limit of a Function

If there is a number, *L*, such that the value of $f(x)$ gets closer and closer to *L* as *x* gets closer to *a*, then *L* is called the limit of $f(x)$ as *x* approaches *a*.

In symbols, this is represented as the following: $L = \lim_{x \to a} f(x)$.

One-Sided Limits

A left-hand limit is represented with the notation $\lim_{x \to a^-} f(x) = L$, and indicates that as *x* approaches *a* from the left, but remains less than *a*, the values of $f(x)$ get closer to *L.*

A right-hand limit is represented with the notation $\lim_{x \to a+} f(x) = L$, and indicates that as *x* approaches *a* from the right, but remains greater than *a*, the values of $f(x)$ get closer to *L.*

Unequal One-Sided Limits

The $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$. In other words, if the left-hand limit and right-hand limit are not the same, then the limit does not exist.

Properties of Limits Lesson

Properties of Limits

Let *a* and *c* be real numbers, and let *k* be a whole number. Assume the $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Limit of a Constant Function	$\lim_{x \to a} c = c$
Limit of the Identity Function	$\lim_{x \to a} x = a$
Limit of a Sum	$\lim_{x \to a} \left(f(x) + g(x) \right) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

Limit of a Difference	$\lim_{x\to a}\left(f(x)-g(x)\right)=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$
Limit of a Product	$\lim_{x\to a}\left(f(x)\cdot g(x)\right)=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$
Limit of a Constant Multiple	$\lim_{x\to a} c\cdot f(x)=c\cdot\lim_{x\to a} f(x)$
Limit of a Quotient	$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \lim_{x\to a} g(x)\neq 0$
Limit of a Power	$\lim_{x\to a} x^k = a^k$

Continuity of Functions and Limits Lesson

Criteria for a Continuous Function

A function, *f*, is continuous at a number, *a*, when three conditions are met:

- The function *f* is defined at *a* so that $f(a)$ is a real number.
- $\lim_{x\to a} f(x)$ exists
- $\lim_{x\to a} f(x)=f(a)$

Rate of Change Lesson

The Derivative of *f* at *a*

The slope of the tangent line to a curve, *f*, at $(a, f(a))$, or the derivative of *f* at *a* is defined by the following:

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

This limit also represents the slope of *f* at $(a, f(a))$ and the instantaneous rate of change of *f* at $(a, f(a))$.

Applications of the Derivative Lesson

Average Velocity

From time *t* = *a* to *t* = *a* + *h*, the change in an object's position is $f(a+h)-f(a)$.

The average velocity, v_{avg}, is defined as the change in position over the change in time.

$$v_{\text{avg}} = \frac{f(a+h)-f(a)}{h}$$

Instantaneous Velocity

The instantaneous velocity of an object at a specific time, *t*, is the limit of the average velocity as *h* approaches 0, or the derivative of the object's position.

$$v(t) = f'(t) = \lim_{h \to 0} \frac{f(t+h)-f(t)}{h}$$

Acceleration

The acceleration of an object at a specific time, *t*, is defined as the rate at which the object's velocity is changing, or the derivative of the object's velocity at that time.

$$a(t) = v'(t) = \lim_{h \to 0} \frac{v(t+h)-v(t)}{h}$$